What is a Logical System 2020*

Dov Gabbay

In honour of Johan van Benthem

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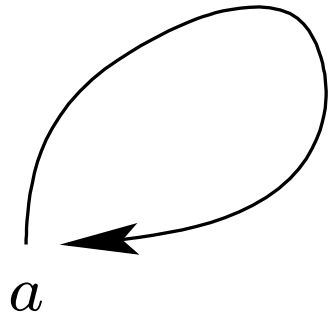
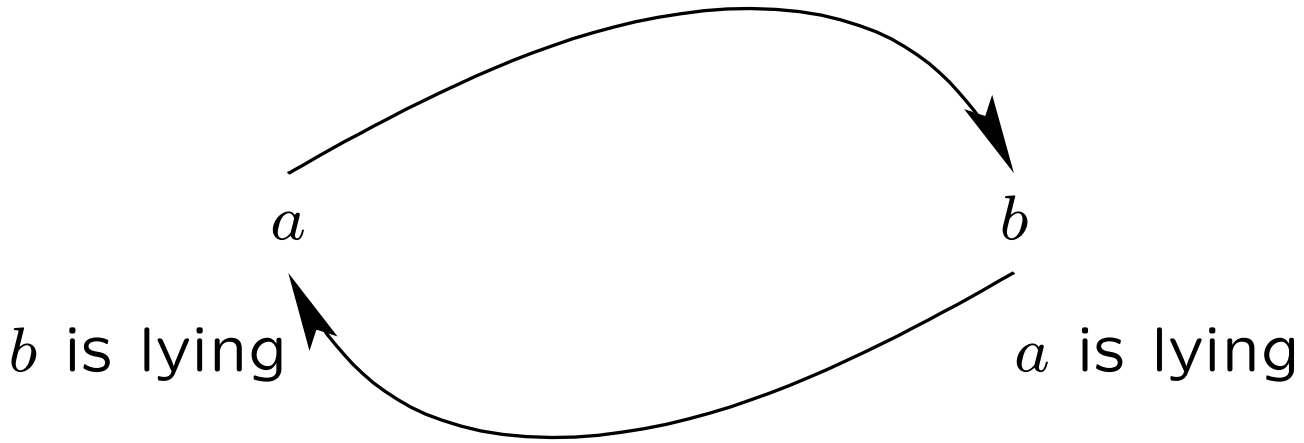
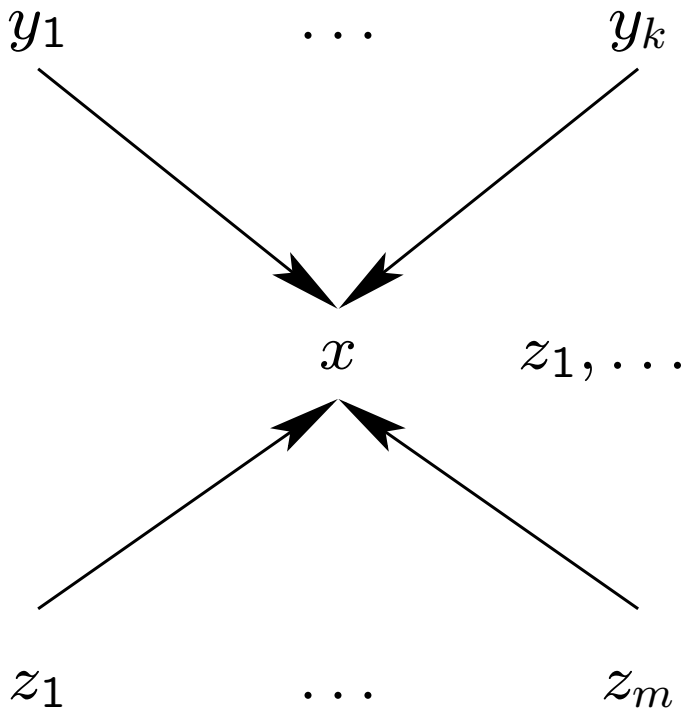


I am lying



x is lying

x is lying



$$(S, R), R \subseteq S \times S$$

$E \subseteq S$ is a complete extension iff

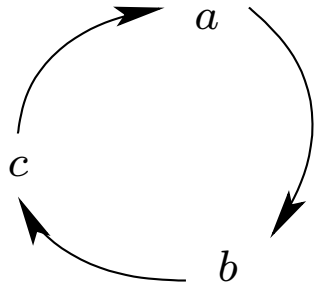
1. Conflict free

$$x, y \in E \Rightarrow \neg xRy.$$

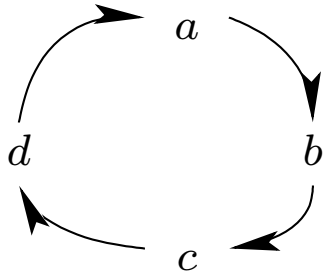
2. Protects itself from liars

$$y \in E \wedge xRy \Rightarrow \exists z \in E zRx.$$

3. Preferred extension:
   E is maximal wrt (1)–(2)



∅ extension



∅

$\{a, c\}$

$\{b, d\}$

Caminada labelling

$\lambda(x) = 1$ if $x \in E$

$\lambda(x) = 0$ if $\exists y \in E(yRx)$

$\lambda(x) = \frac{1}{2}$ otherwise

Equational approach

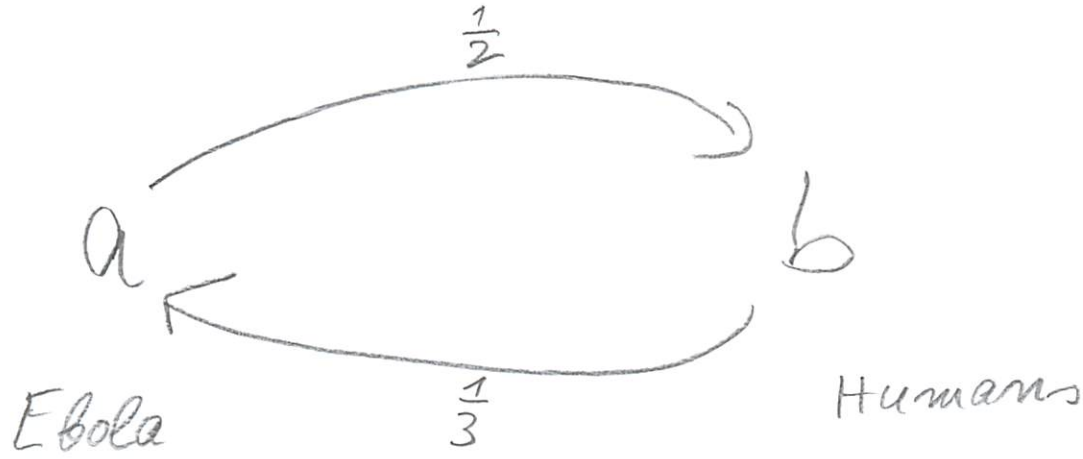
(S, R) is a bearer of equations.

$$Eq_{\mathsf{max}} : x = 1 - \mathsf{max}\{y_i\}$$
$$Eq_{\mathsf{inv}} : x = \Pi_i(1 - y_i)$$
$$\{y_i\} = \{y | yRx\}.$$

Solutions to the Eq_{max} equations give all extensions E

Solutions to the Eq_{inv} equations give all preferred extensions and some more.

$$E = \{x | f(x) = 1\}, \{x | f(x) = 0\} = \{x | \exists y \in E yRx\}.$$



Ebola can kill $\frac{1}{2}$ Human race

Humans can kill $\frac{1}{3}$ Ebola.

$$a = 1 - \frac{1}{3}b$$

$$b = 1 - \frac{1}{2}a$$

steady state : $a = 0.8$ $b = 0.6$

Gabbay–Rodrigues iteration schema for Eq_{max}

Given initial values $V_0(x) x \in S$.

$$V_{i+1}(x) = (1 - V_i(x)) \min(\tfrac{1}{2}, 1 - \max_{yRx}\{V_i(y)\}) + V_i(x) \max(\tfrac{1}{2}, 1 - \max_{yRx}\{V_i(y)\})$$

$V_\infty(x) = \lim V_i(x)$, gives a solution

Important: Stability. Given V_0, we get best compatible solution.

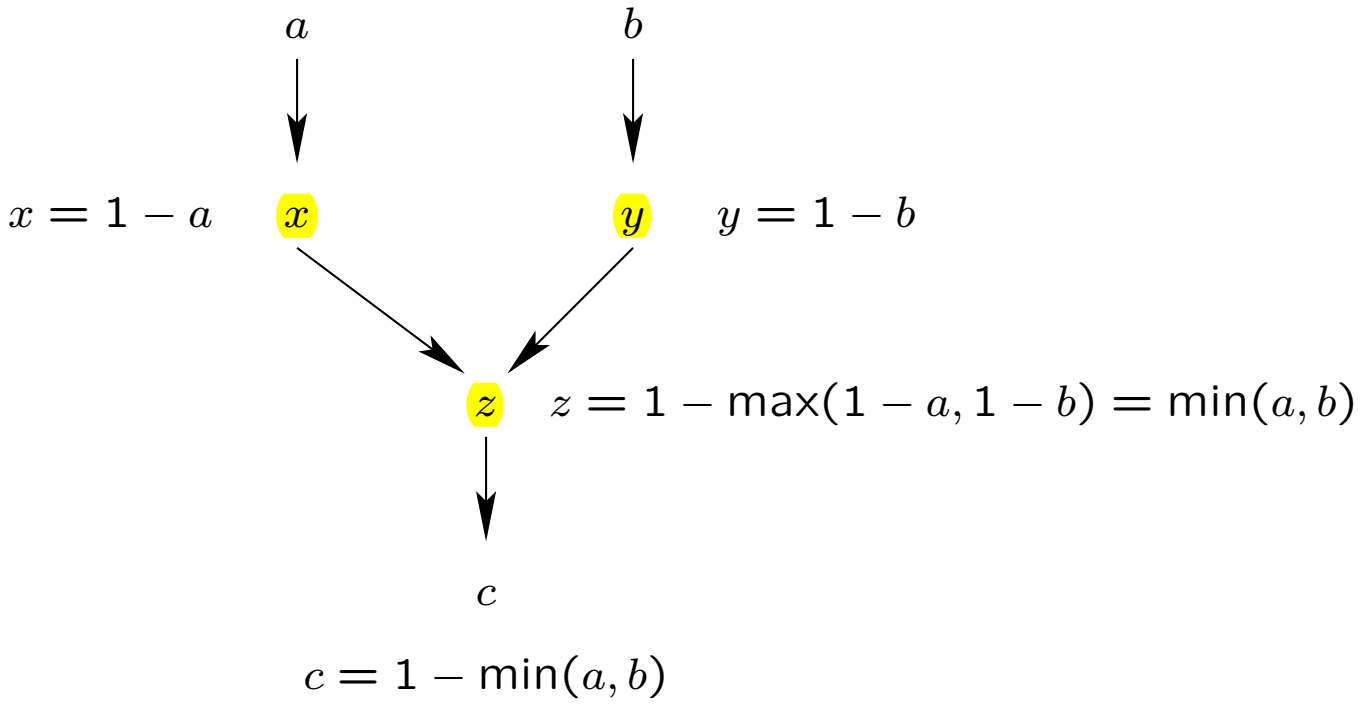
What is a logic 2020?

1. Express logics using movements on network.

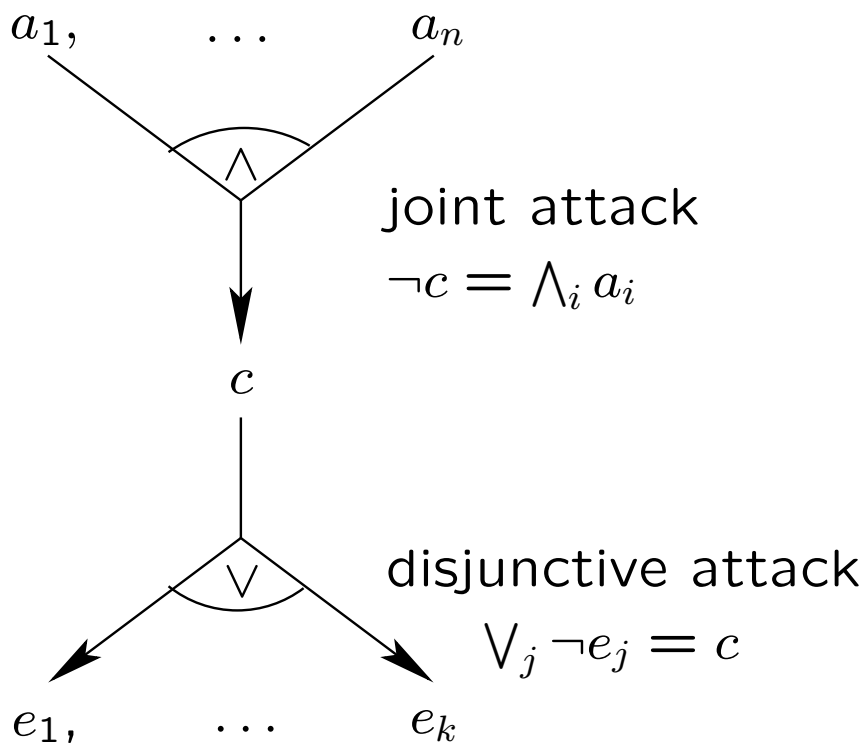
2. Express (1) using equations.

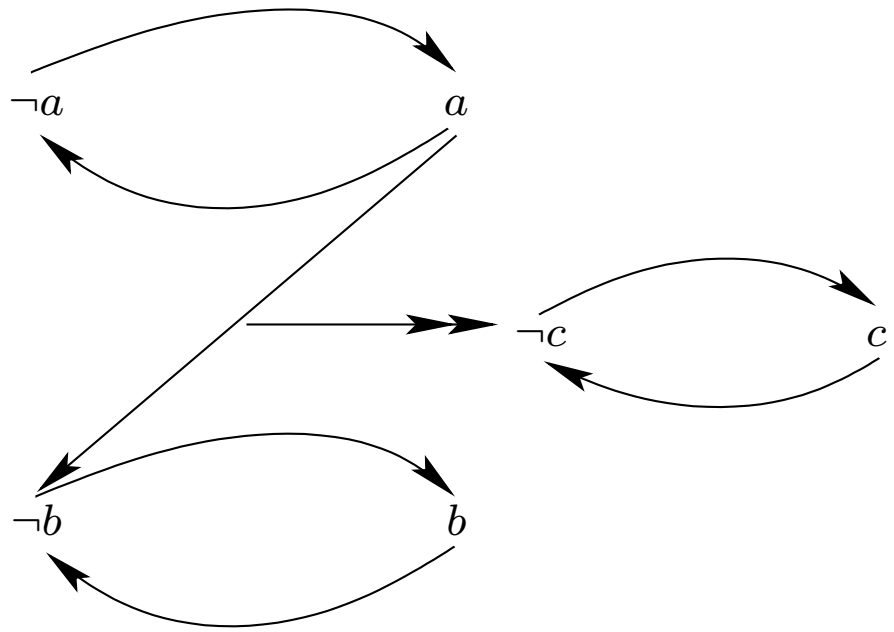
3. Compile equations in Matlab.

4. Logic = Agent = Equations.



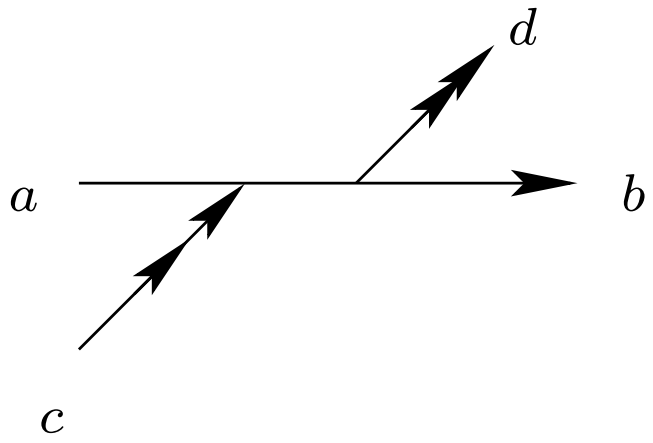
Note that x, y, z do not appear in the equation


$a_1,$
$\ldots$
a_n
$\bigwedge$
joint attack
$\neg c = \bigwedge_i a_i$
c
$\bigvee$
disjunctive attack
$\bigvee_j \neg e_j = c$
$e_1,$
$\ldots$
e_k

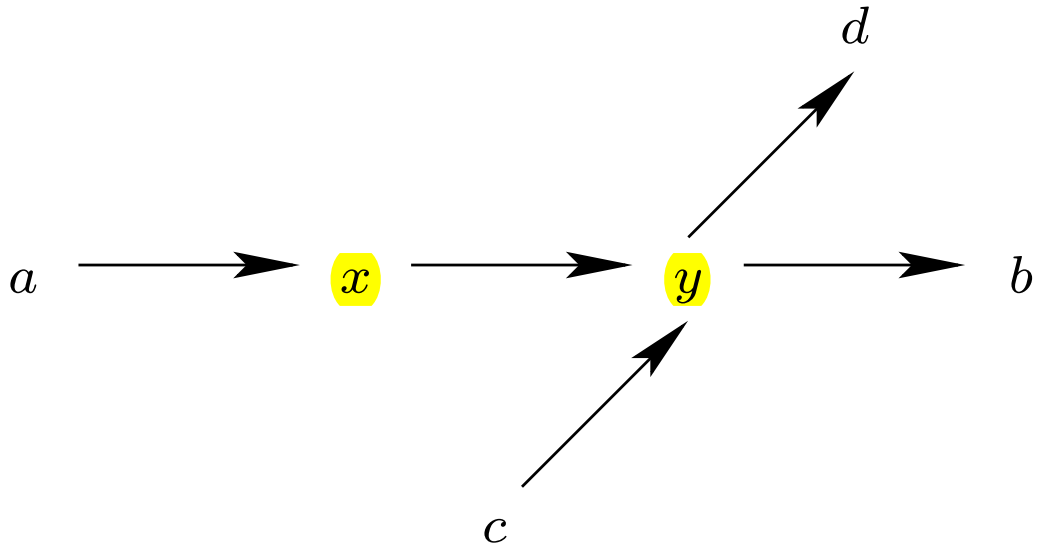


$(a \Rightarrow b) = (a$ implies $b) = (a$ attacks $\neg b)$

$((a \Rightarrow b) \Rightarrow c) = (a \rightarrow \neg b) \twoheadrightarrow \neg c)$

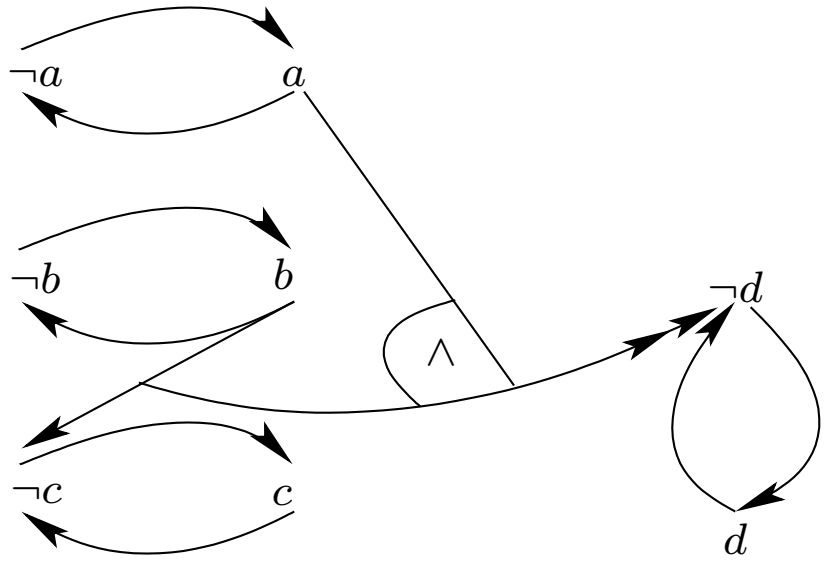


$$c \twoheadrightarrow (a \longrightarrow b) \twoheadrightarrow d$$



Goal directed clause can define many logics

$$a \wedge (b \Rightarrow c) \Rightarrow d$$



Deduction is an algorithm for moving around the network collecting nodes to form an extension.

- Extension = model
- Set of nodes = theory.

References

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