Natural Logic and Vehicles of Inference
Celebration Event in Honor of Johan van Benthem

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Natural Logic (van Benthem, 1987):

“using linguistic constructs directly as a vehicle of inference”
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“using linguistic constructs directly as a vehicle of inference”

rather than focusing on a single “worst-case vehicle of interpretation,” we should recognize “a variety of such interpretations, whose structure is itself a topic for semantic research.”
Classic example: Monotonicity Calculus

Every American likes jazz  
\[\text{Every Tennessean likes jazz}\]  
\[\text{Every Tennessean likes some form of music}\]
Classic example: Monotonicity Calculus

\[
\begin{align*}
\text{Every American likes jazz} \\
\text{Every Tennessean likes jazz} \\
\text{Every Tennessean likes some form of music}
\end{align*}
\]
Classic example: Monotonicity Calculus

\[
\text{No American likes jazz} \\
\text{No Tennessean likes jazz} \\
\text{No Tennessean likes West Coast jazz}
\]

\[\downarrow \text{No} \downarrow\]
Classic example: Monotonicity Calculus

\[
\downarrow \text{Every} \uparrow \quad \downarrow \text{No} \downarrow
\]
Classic example: Monotonicity Calculus

\[
\begin{align*}
\downarrow \text{Every} & \uparrow \\
\downarrow \text{No} & \downarrow \\
\uparrow \text{Some} & \uparrow \\
\uparrow \text{Not}_\text{Every} & \downarrow
\end{align*}
\]
Classic example: Monotonicity Calculus

↓ Every ↑

↓ No ↓

↑ Some ↑

↑ Not-Every ↓

∗ Most ↑

∗ Few ↓
Frank failed to complete his taxes

Frank failed to complete his taxes on time
Frank failed to complete his taxes

Frank failed to complete his taxes on time

Every Tennessean who failed to complete his taxes on time likes jazz

Every Tennessean who failed to complete his taxes likes jazz
Every [American]$^-$ likes [jazz]$^+$

Every [Tennessean]$^-$ likes [jazz]$^+$

Every [Tennessean]$^-$ likes [some form of music]$^+$

Frank failed to [complete his taxes]$^-$

Frank failed to [complete his taxes on time]$^-$

Every Tennessean who failed to [complete his taxes on time]$^+$ likes jazz
All Americans like [jazz]$^+$

All Americans likes [some form of music]$^+$

Most Americans likes [jazz]$^+$

Most Americans likes [some form of music]$^+$

From a logical point of view, such patterns cross-cut the standard “order hierarchy.”

Empirically, these two argument patterns are indeed seen to be equally easy for people (Oaksford & Chater, 2001).
All Americans like [jazz]+

All Americans likes [some form of music]+

Most Americans likes [jazz]+

Most Americans likes [some form of music]+

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All Americans like [jazz]

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Most Americans likes [some form of music]

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1. Natural logic as opening up new research questions in pure logic: “logic for its own sake”

2. Deep links between logic and other areas of inquiry, in this case language and cognition.

This dual influence is characteristic of Johan’s work.
Part 1: Logical Issues
Early work on Monotonicity Calculus by van Benthem (1986) and Sánchez-Valencia (1991) showed how to mark types with monotonicity information and develop simple proof systems.
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Soundness of the marking system, and hence of the proof system, was established. However, completeness remained open.
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(van Benthem 2011, CSLI Handout)

In recent work, Larry Moss and I have taken on this question, couching it in a broader study of monotonicity reasoning in general (2013, 2014).
Simple Arithmetic Example

Which is bigger, \(-(7 + 2^{-3})\) or \(-(7 + 2^{-4})\)?
Simple Arithmetic Example

Which is bigger, $-(7 + 2^{-3})$ or $-(7 + 2^{-4})$?

\[
\begin{align*}
3 &< 4 \\
\frac{-4}{-3} &< 1 \\
\frac{2^{-4}}{2^{-3}} &< 1 \\
\frac{7 + 2^{-4}}{7 + 2^{-3}} &< 1 \\
-(7 + 2^{-3}) &< -(7 + 2^{-4})
\end{align*}
\]
Types and Type Domains

Definition
\[ M = \{ +, -, \cdot \}. \]
Types and Type Domains

Definition
$\mathcal{M} = \{+,-,\cdot\}$.

Definition
Let $\mathcal{B}$ be a set of base types. The full set of types $\mathcal{T}$ is defined as the smallest superset of $\mathcal{B}$, such that whenever $\sigma, \tau \in \mathcal{T}$, so is $\sigma \xrightarrow{m} \tau$, for each $m \in \mathcal{M}$. 
Definition

A *structure* is a system $\mathcal{S} = \{ \mathbb{D}_\tau \}_{\tau \in \mathcal{T}}$ of preorders.

Base types $\beta \in \mathcal{B}$ have no requirement on $\mathbb{D}_\beta$. 
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For functional types $\sigma \xrightarrow{m} \tau$, we require:

1. $D_{\sigma \rightarrow \tau}^+ = \text{monotone functions from } \mathbb{D}_\sigma \text{ to } \mathbb{D}_\tau$.
2. $D_{\sigma \rightarrow \tau}^- = \text{antitone functions from } \mathbb{D}_\sigma \text{ to } \mathbb{D}_\tau$.
3. $D_{\sigma \rightarrow \tau} \cdot = \text{all functions from } \mathbb{D}_\sigma \text{ to } \mathbb{D}_\tau$.
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The ordering $\leq_{\sigma \xrightarrow{m} \tau}$ on $D_{\sigma \xrightarrow{m} \tau}$ is given pointwise:

$f \preceq_{\sigma \xrightarrow{m} \tau} g$ if and only if $f(a) \leq_{\tau} g(a)$ for $a \in D_\sigma$. 
There is a natural relation $\leq$ between types, such that whenever $\sigma \leq \tau$, there is a canonical embedding from $D_\sigma$ into $D_\tau$. 
Definition
Given a set $\text{Con}$ of constants and a function \(\text{type} : \text{Con} \rightarrow \mathcal{T}\), the set $\mathcal{T}$ of typed terms $t : \tau$ is defined recursively, as follows:
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Given a set \( \text{Con} \) of constants and a function \( \text{type} : \text{Con} \rightarrow \mathcal{T} \), the set \( \mathcal{T} \) of typed terms \( t : \tau \) is defined recursively, as follows:

1. If \( c \in \text{Con} \), then \( c : \text{type}(c) \) is a typed term.

2. If \( t : \sigma \xrightarrow{m} \tau \) and \( u : \rho \) are typed terms and \( \rho \preceq \sigma \), then \( t(u) : \tau \) is a typed term.
Monotonicity Calculus

(Refl) \( t \leq t \)

(Trans) \( t \leq u \quad u \leq v \rightarrow t \leq v \)

(Mono) \( u \leq v \rightarrow t^{\uparrow}(u) \leq t^{\uparrow}(v) \)

(Anti) \( v \leq u \rightarrow t^{\downarrow}(u) \leq t^{\downarrow}(v) \)

(Point) \( s \leq t \rightarrow s(u) \leq t(u) \)
Monotonicity Calculus

\( (\text{REFL}) \quad \frac{t \leq t}{t \leq t} \)

\( (\text{TRANS}) \quad \frac{t \leq u \quad u \leq v}{t \leq v} \)

\( (\text{MONO}) \quad \frac{u \leq v}{t_{\uparrow}(u) \leq t_{\uparrow}(v)} \)

\( (\text{ANTI}) \quad \frac{v \leq u}{t_{\downarrow}(u) \leq t_{\downarrow}(v)} \)

\( (\text{POINT}) \quad \frac{s \leq t}{s(u) \leq t(u)} \)

Theorem (Icard & Moss, 2013)

This calculus is sound and strongly complete.
We can also include variables and \( \lambda \)-abstraction.
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As Johan long ago observed, this requires a kind of preservation result.
Lemma (Icard & Moss)

1. $\lambda x.t$ is semantically monotone iff all free occurrences of $x$ in $t$ are in positive position.

2. $\lambda x.t$ is semantically antitone iff all free occurrences of $x$ in $t$ are in negative position.
(\textsc{Refl}) \frac{\text{Reflexivity}}{t \leq t} \quad \quad \quad (\textsc{Trans}) \quad \frac{t \leq u \quad u \leq v}{t \leq v}

(\textsc{Mono}) \quad \frac{u \leq v}{\uparrow(t(u)) \leq \uparrow(t(v))} \quad \quad (\textsc{Anti}) \quad \frac{v \leq u}{\downarrow(t(u)) \leq \downarrow(t(v))}

(\textsc{Point}) \quad \frac{s \leq t}{s(u) \leq t(u)}
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(Point) \[ \frac{s \leq t}{s(u) \leq t(u)} \]

(\alpha) \[ \frac{\lambda x.t \leq \lambda y.t^x_y}{(\lambda x.t)s \equiv t^x_s} \]

(\beta) \[ \frac{t \leq s}{\lambda x.t \leq \lambda x.s} \]
Theorem (Icard & Moss, 2014)
This calculus is sound and strongly complete.

N.B. It is undecidable, if assumptions are allowed!
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In this setting—with a distinguished truth-value type—several open questions remain.
Preservation Question

- Suppose $t$ is of truth value type. When is $\lambda x.t$ semantically monotone or antitone?
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Preservation Question

- Suppose $t$ is of truth value type. When is $\lambda x.t$ semantically monotone or antitone?
- For the so called Lambek fragment of $\lambda$-calculus, van Benthem (1991) showed that this can be characterized in terms of positive and negative occurrences.
- For the general case this is open.
Further Logical Issues

- Systems with “internalized polarity” marking, and connections with negative polarity items (Dowty 1994; Bernardi 2002; Moss 2012; etc.).
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- Beyond monotonicity, including other logical relations and classes of functions (MacCartney & Manning 2008, 2009; Icard 2012, 2014).
Part 2: Application to Language and Cognition
The Monotonicity Calculus has been influential in computational linguistics and psycholinguistics:

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▶ Suggestive results by Geurts (2003) and Geurts & van der Slik (2005) provide evidence that monotonicity plays an important role in language processing.
“Even before disambiguation has taken place, some consequences can usually be drawn already. Inference is not an all-or-nothing matter.”

(van Benthem 1987)
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1. It would be computationally expensive to default always to the most complex, “worst case” interpretation.
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1. It would be **computationally expensive** to default always to the most complex, “worst case” interpretation.

   Everyone talked with at least four people from different departments
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2. There may be uncertainty about the language itself, whereas certain inferences do not even require disambiguation.

   Most who know a foreign language learned it at home
   Most who know a foreign language learned it at home or at school
But . . . as van Benthem (1987) points out, one must be careful with surface reasoning:

Everyone with a garden water it

Everyone with a garden-statue water it
Question: What does it mean to “use linguistic constructs directly as a vehicle of inference”? 

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Suppose we randomly sample situations in which a premise $t(u)$ is true, and $v$ for which either $u \leq v$ or $v \leq u$, in order to determine whether $t(v)$ follows.
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How likely are we to be correct?

- If it does follow, we will always be correct.
- If not, it depends on what the terms are . . .
Some\((A, C)\) \quad B \subseteq C
\hline
 Some\((A, B)\)
How “dense” are the counterexamples to these inference patterns among “small” models?
Some($A$, $C$) $B \subseteq C$ \quad \frac{\text{Some}(A, B)}{\text{All}(C, B)} \quad C \subseteq A \quad \frac{\text{All}(A, B)}{\text{All}(C, B)}$

How “dense” are the counterexamples to these inference patterns among “small” models?
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- Some: 3 random models of size 3
- All: 4 random models of size 4
- All ∗: 3 random models of size 3
- Most: 7 random models of size 16
- Most ∗: 3 random models of size 5
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Why is it so hard to find small countermodels for inferences of the following form?

\[
\begin{align*}
\text{Most}(C, B) & \quad A \subseteq C \\
\hline
\text{Most}(A, B)
\end{align*}
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Because often enough, this is a plausible inference!
Why is it so hard to find small countermodels for inferences of the following form?

\[
\begin{align*}
\text{Most}(C, B) & \quad A \subseteq C \\
& \Rightarrow \quad \text{Most}(A, B)
\end{align*}
\]

Because often enough, this is a plausible inference!

(Cf. fact that NPIs appear in restrictor of \textit{Most}.)

T. F. Icard: Natural Logic and Vehicles of Inference, Celebration Event in Honor of Johan van Benthem
Moral of this quick exercise:

- For the classic parts of Monotonicity Calculus, random model checking can be very effective.
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- Natural Logic, and especially Johan’s own discussion of it, points the way toward potentially useful coarsenings for this purpose:
Moral of this quick exercise:

▶ For the classic parts of Monotonicity Calculus, random model checking can be very effective.

▶ This also allows going beyond logical validity, to assess (quickly and efficiently) merely plausible conclusions.

▶ Natural Logic, and especially Johan’s own discussion of it, points the way toward potentially useful coarsenings for this purpose: perhaps these are the vehicles of inference.
Recall two quotations:

“a variety of interpretations, whose structure is itself a topic for semantic research”

“some ‘rougner semantics’, closer to mental models that we use . . . ”
Conclusion

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- I believe it holds yet further promise, as we think harder about how these vehicles of inference should be characterized, which in turn opens up new interfaces with cognitive science, e.g., via probabilistic computation.
Thanks for listening!

And thank you, Johan, for all the inspiring ideas (and much else besides)!