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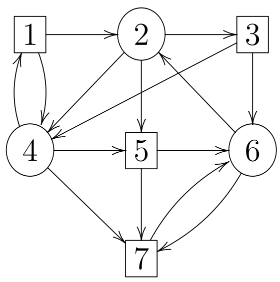
Homework Exercises for the Course Automata Theoretic Foundations of Infinite Games of the KNAW Master Class, Amsterdam, February 2007

Note: This is a choice of exercises. See the KNAW website for information about the required number of solutions.

Exercise 1
Consider the following game graph:



Let the winning conditions for Player B be

(a) $|Occ(\rho))| \leq 2$, and

(b) $|Occ(\rho))| \leq 3$.

Find for each winning condition the winning region for Player B and describe (informally) a winning strategy. Hint: positional strategies suffice.

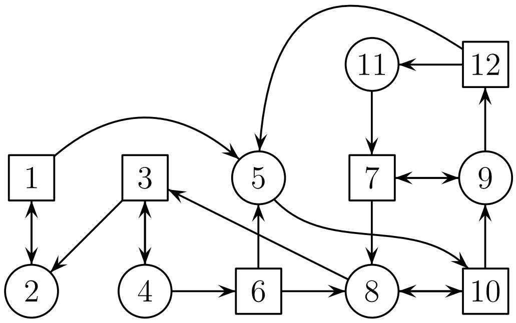
Exercise 2
Consider the game graph of Exercise 1 with the winning condition for plays π:

$$\mathrm{Occ}(\pi) = \{1, \ldots, 7\}.$$

(a) Find the winning region for Player B and describe a winning strategy.

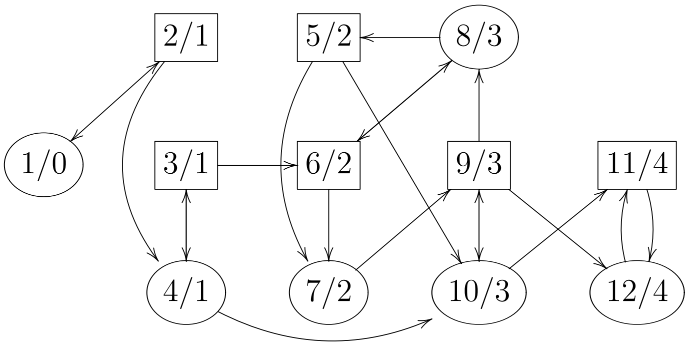
(b) Show that there is no positional winning strategy for Player B in this game.

Exercise 3
Consider the following reachability game with $F = \{1, 2, 10\}$.



Compute the sets $\mathrm{Attr}_B^i(F)$ and deduce the winning regions for both players together with their positional winning strategies.

Exercise 4
Consider the following weak parity game, where vertex i with color j is denoted by i/j.



Compute the winning regions, and the corresponding positional winning strategies, for Player B and A.

Exercise 5 *
We consider game graphs of the form $(G, (F_1, F_2))$ where $F_1, F_2 \subseteq Q$ together with the following winning condition for Player B:

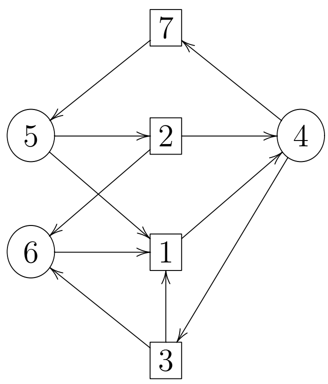
$$\pi \in \mathrm{Win} \Leftrightarrow \mathrm{Occ}(\pi) \cap F_1 \neq \emptyset \text{ and } \mathrm{Occ}(\pi) \cap F_2 \neq \emptyset.$$

For the following two sets prove or disprove that they define the winning region of Player B in such games.

- $U := \mathrm{Attr}_B(F_1 \cap \mathrm{Attr}_B(F_2)) \cup \mathrm{Attr}_B(F_2 \cap \mathrm{Attr}_B(F_1))$
- $V := \mathrm{Attr}_B\left((F_1 \cap \mathrm{Attr}_B(F_2)) \cup (F_2 \cap \mathrm{Attr}_B(F_1))\right)$

Exercise 6

Consider the game graph G on the right, and the Muller condition $\mathcal{F} = \{\{2,4,5,7\}, \{1,2,3,4,5,6,7\}\}$.
Find an automaton winning strategy for Player B in the Muller game with as few states as possible, and show that any other automaton strategy with less states is not winning for Player B.



Exercise 7
Consider the DJW game over the graph with vertices $\{A, B, C, D, 1, 2, 3, 4\}$ as given in the lecture. We propose the following *latest appearance queue (LAQ)* strategy for Player B, initializing the queue with ABCD.

- Add the current vertex at the front of the LAQ, delete the last vertex.
- Move to the vertex whose number is the number of different vertices in the current LAQ.

The following table shows the evolution of the LAQ for an example sequence of visited letter states:

	A	C	C	D	B	D	C	D	D	...
ABCD	AABC	CAAB	CCAA	DCCA	BDCC	DBDC	CDBD	DCDB	DDCD	...

Decide whether the new LAQ strategy is a winning strategy for Player B. Prove this or give a counter-example.

Exercise 8 *
Let $G = (Q, E)$ with $Q = Q_A \uplus Q_B$ a be game graph. A *nondeterministic positional strategy (NPS)* for Player B in G is a relation $R \subseteq (Q_B \times Q) \cap E$. A play $q_0, q_1, \dots$ is played according to R if for all $q_i \in Q_0$ we have $(q_i, q_{i+1}) \in R$. Let φ be a winning condition. An NPS R is a nondeterministic winning strategy for player B from q if all plays from q played according to R satisfy φ.
Decide whether the union of two winning NPS from q is again a winning NPS from q for:

(a) reachability conditions ($\varphi = \exists i\ \rho(i) \in F$).

(b) safety conditions ($\varphi = \forall i\ \rho(i) \notin F$).

Exercise 9 *
Let $G = (Q, E)$ be a game graph. A *request response* condition for G is given by two sets $\mathrm{Req}, \mathrm{Resp} \subseteq Q$. Player B wins a play ρ in the *request response* game $(G, \mathrm{Req}, \mathrm{Resp})$ iff

$$\forall n, \rho(n) \in \mathrm{Req} \rightarrow \exists m > n,\ \rho(m) \in \mathrm{Resp}$$

Find a reduction of request response games to parity games by introducing suitable memory component and colors. Discuss the question whether you can decrease the number of colors of your solution.

Exercise 10
Consider the parity tree automaton $\mathcal{A} = (\{q_0, q_1, q_2, q_3\}, q_0, \Delta, \Omega)$ running over trees labeled by $\Sigma = \{a, b\}$ whose the set of transitions is given below:

$$(q_0, a, q_0, q_1) \quad (q_1, a, q_1, q_1) \quad (q_1, b, q_2, q_2) \quad (q_2, b, q_2, q_2)$$

The priority of q_i is i. Take your patience to construct the associated input-free automaton $\mathcal{A}'$ with states in $Q \times \Sigma$ and the finite parity game $G_{\mathcal{A}'}$ associated to $\mathcal{A}'$. Use this game to decide the emptyness of $L(\mathcal{A})$ and to give a regular tree belonging to $L(\mathcal{A})$.