

# Experimental Validation of Quantum Game Theory

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(Dated: June 27, 2008)

This article uses data from two experimental studies of two-person prisoner's dilemma games [1, 2] and compares the data with the theoretic predictions calculated with the use of a quantum game theoretical method. The experimental findings of the cooperation percentage indicate a strong connectivity with the properties of a novel function, which depends on the payoff parameters of the game and on the value of entanglement of the players' strategies. A classification scheme depending on four quantum cooperation indicators is developed to describe cooperation in real two-person games. The quantum indicators lead to results, which are more precise than the cooperation predictions derived from classical game theory.

PACS numbers: 01.20.+x, 01.50.Pa, 02.50.Le, 03.67., 89.20.-a, 89.65.-s, 89.70.+c, 89.75.Fb

## INTRODUCTION

Quantum game theory (QGT)[32] has its origin in elementary particle physics and quantum information theory. In 1999 the first formulations of quantum game theory were presented by D. A. Meyer [3] and J. Eisert et al. [4]. Unknowing Meyers' results on the "Penny Flip" game, Eisert and colleagues focused on the prisoner's dilemma game. Within their quantum representation they were able to demonstrate that prisoners could escape from the dilemma, if the entanglement of the two-person quantum wave function lies above a certain value. In 2001 J. Du et al. [5] realized the first simulation of a quantum game on their nuclear magnetic resonance quantum computer. Later, in 2007 A. Zeilinger et al. accomplished a quantum game on a one-way quantum computer [6]. The application of quantum game theory to an existing social system, namely the publication network of scientists, was presented in M. Hanauske et al. [7]. The authors showed, that quantum game theory could give a possible explanation of the differing publishing methods of scientific communities. A validation of quantum game theoretical concepts by using experimental data of real two-person games was addressed in K.-Y. Chen and T. Hogg [8] (see also [9]). In contrast to the experimental data used in the present article, the authors of [8] used an experimental design, which includes a quantum version of the game. Our understanding of an inclusion of quantum strategies in the players' decisions is different, insofar as we interpret the whole process of a real game as a quantum game.

In this paper, on the one hand, we develop cooperation indicators derived from a quantum game theoretical approach, and on the other hand we address the following

research question: *Compared to cooperation indicators based on classical game theory, how precise do "quantum" indicators predict the outcome of real person game experiments.*

Based on Eisert's two-player quantum protocol [4] and the concept of quantum Nash equilibria, four quantum cooperation indicators are defined. By using these indicators to predict the cooperation rates of real two-person games it will be shown that the quantum indicators lead to results, which are at least as good as the cooperation predictions derived from classical game theory.

The present article is structured as follows: After presenting the main mathematical formulations used within the quantum game theoretical approach the "quantum" cooperation indicators are defined, visualized and compared with the classical indicators. Afterwards we address our research question and present the experimental validation of the cooperation predictions derived from quantum game theory. The paper ends with a short summary of the main findings.

## MATHEMATICS OF QGT

The normal-form representation of a two-player game  $\Gamma$ , where each player (Player 1  $\hat{=}$  A, Player 2  $\hat{=}$  B) can choose between two strategies ( $\mathcal{S}^A = \{s_1^A, s_2^A\}$ ,  $\mathcal{S}^B = \{s_1^B, s_2^B\}$ ) is the classical grounding of the two-player quantum game focused on in this article. In our case the two strategies represent the players' choice between cooperating (not confess, C) or defecting (confess, D) in a prisoner's dilemma game. The whole strategy space  $\mathcal{S}$  is composed with use of a Cartesian product of the individual strategies of the two players:

$$\mathcal{S} = \mathcal{S}^A \times \mathcal{S}^B = \{(C,C), (C,D), (D,C), (D,D)\} \quad (1)$$

The payoff structure of a prisoner's dilemma game can be described by the following matrix:

A \ B	C	D
C	(c,c)	(a,b)
D	(b,a)	(d,d)

TABLE I: General prisoner's dilemma payoff matrix.

The parameters a, b, c, and d should satisfy the following inequations [1, 10]

$$b > c > d > a, \quad 2c > a + b \quad (2)$$

In quantum game theory, the measurable classical strategies (C and D) correspond to the orthonormal unit basis vectors  $|C\rangle$  and  $|D\rangle$  of the two dimensional complex space  $\mathbb{C}^2$ , the so called Hilbert space  $\mathcal{H}_i$  of the player  $i$  ( $i = A, B$ ). A quantum strategy of a player  $i$  is represented as a general unit vector  $|\psi\rangle_i$  in his strategic Hilbert space  $\mathcal{H}_i$ . The whole quantum strategy space  $\mathcal{H}$  is constructed with the use of the direct tensor product of the individual Hilbert spaces:  $\mathcal{H} := \mathcal{H}_A \otimes \mathcal{H}_B$ . The main difference between classical and quantum game theory is that in the Hilbert space  $\mathcal{H}$  correlations between the players' individual quantum strategies are allowed, if the two quantum strategies  $|\psi\rangle_A$  and  $|\psi\rangle_B$  are entangled. The overall state of the system we are looking at is described as a two-player quantum state  $|\Psi\rangle \in \mathcal{H}$ . We define the four basis vectors of the Hilbert space  $\mathcal{H}$  as the classical game outcomes ( $|CC\rangle := (1, 0, 0, 0)$ ,  $|CD\rangle := (0, -1, 0, 0)$ ,  $|DC\rangle := (0, 0, -1, 0)$  and  $|DD\rangle := (0, 0, 0, 1)$ ).

The setup of the quantum game begins with the choice of the initial state  $|\Psi_0\rangle$ . We assume that both players are in the state  $|C\rangle$ . The initial state of the two players is given by

$$|\Psi_0\rangle = \hat{\mathcal{J}}|CC\rangle = \begin{pmatrix} \cos(\frac{\gamma}{2}) \\ 0 \\ 0 \\ i \sin(\frac{\gamma}{2}) \end{pmatrix}, \quad (3)$$

where the unitary operator  $\hat{\mathcal{J}}$  (see equation 8) is responsible for the possible entanglement of the two-player system. The players' quantum decision (quantum strategy) is formulated with the use of a two parameter set of uni-

tary  $2 \times 2$  matrices:

$$\hat{U}(\theta, \varphi) := \begin{pmatrix} e^{i\varphi} \cos(\frac{\theta}{2}) & \sin(\frac{\theta}{2}) \\ -\sin(\frac{\theta}{2}) & e^{-i\varphi} \cos(\frac{\theta}{2}) \end{pmatrix} \quad (4)$$

$$\forall \theta \in [0, \pi] \wedge \varphi \in [0, \frac{\pi}{2}]$$

By arranging the parameters  $\theta$  and  $\varphi$ , a player chooses his quantum strategy. The classical strategy C is selected by appointing  $\theta = 0$  and  $\varphi = 0$ :

$$\hat{C} := \hat{U}(0, 0) = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad (5)$$

whereas the strategy D is selected by choosing  $\theta = \pi$  and  $\varphi = 0$ :

$$\hat{D} := \hat{U}(\pi, 0) = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (6)$$

In addition, the quantum strategy  $\hat{Q}$  is given by

$$\hat{Q} := \hat{U}(0, \pi/2) = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \quad (7)$$

After the two players have chosen their individual quantum strategies ( $\hat{U}_A := \hat{U}(\theta_A, \varphi_A)$  and  $\hat{U}_B := \hat{U}(\theta_B, \varphi_B)$ ) the disentangling operator  $\hat{\mathcal{J}}^\dagger$  is acting to prepare the measurement of the players' state. The entangling and disentangling operator ( $\hat{\mathcal{J}}, \hat{\mathcal{J}}^\dagger$ ; with  $\hat{\mathcal{J}} \equiv \hat{\mathcal{J}}^\dagger$ ) is depending on one additional single parameter  $\gamma$  which measures the strength of the entanglement of the system:

$$\hat{\mathcal{J}} := e^{i\frac{\gamma}{2}(\hat{D} \otimes \hat{D})}, \quad \gamma \in [0, \frac{\pi}{2}]. \quad (8)$$

The entangling operator  $\hat{\mathcal{J}}$  in the used representation has the following explicit structure:

$$\hat{\mathcal{J}} := \begin{pmatrix} \cos(\frac{\gamma}{2}) & 0 & 0 & i \sin(\frac{\gamma}{2}) \\ 0 & \cos(\frac{\gamma}{2}) & -i \sin(\frac{\gamma}{2}) & 0 \\ 0 & -i \sin(\frac{\gamma}{2}) & \cos(\frac{\gamma}{2}) & 0 \\ i \sin(\frac{\gamma}{2}) & 0 & 0 & \cos(\frac{\gamma}{2}) \end{pmatrix} \quad (9)$$

Finally, the state prior to detection can therefore be formulated as follows:

$$|\Psi_f\rangle = \hat{\mathcal{J}}^\dagger (\hat{U}_A \otimes \hat{U}_B) \hat{\mathcal{J}} |CC\rangle \quad (10)$$

The expected payoff within a quantum version of a general two-player game, depends on the payoff matrix (see Table I) and on the joint probability to observe the four

observable outcomes  $P_{CC}, P_{CD}, P_{DC}$  and  $P_{DD}$  of the game:

$$\$A = cP_{CC} + aP_{CD} + bP_{DC} + dP_{DD} \quad (11)$$

$$\$B = cP_{CC} + bP_{CD} + aP_{DC} + dP_{DD}$$

$$\text{with: } P_{\sigma\sigma'} = |\langle \sigma\sigma' | \Psi_f \rangle|^2, \quad \sigma, \sigma' = \{C, D\} \quad .$$

It should be pointed out here, that an entangled two-player quantum state does not mean at all that the persons themselves (or even the players' brains) are entangled. The process of quantum decoherence, with its quantum to classical transition, forbid such macroscopic entangled systems established from microscopic quantum particles [11, 12]. However, peoples' cogitations, represented as quantum strategies, could be associated within an abstract space. Although no measurable accord is present between the players' strategy choices, the imaginary parts of their strategy wave functions might interact, if their individual states are entangled. This quantum phenomenon might possibly be interpreted as the ability of a player to empathize into the other players thinking lanes, which may be originated from similar historical or cultural background. Players with strongly entangled strategies appear to act more like a collective state.

## QUANTUM COOPERATION INDICATORS

Dominant quantum strategies and quantum Nash equilibria are formulated as follows:

$(\theta_A^*, \varphi_A^*; \theta_B^*, \varphi_B^*)$  is a dominant quantum strategy, if

$$\$A(\hat{U}_A^*, \hat{U}_B) \geq \$A(\hat{U}_A, \hat{U}_B) \quad \forall \hat{U}_A \wedge \hat{U}_B \quad (12)$$

$$\$B(\hat{U}_A, \hat{U}_B^*) \geq \$B(\hat{U}_A, \hat{U}_B) \quad \forall \hat{U}_A \wedge \hat{U}_B \quad .$$

$(\theta_A^*, \varphi_A^*; \theta_B^*, \varphi_B^*)$  is a quantum Nash equilibrium, if

$$\$A(\hat{U}_A^*, \hat{U}_B^*) \geq \$A(\hat{U}_A, \hat{U}_B^*) \quad \forall \hat{U}_A \quad (13)$$

$$\$B(\hat{U}_A^*, \hat{U}_B^*) \geq \$B(\hat{U}_A^*, \hat{U}_B) \quad \forall \hat{U}_B \quad .$$

We define the novel function  $\mathcal{N}_A$  of player A in a two-player quantum game by

$$\begin{aligned} \mathcal{N}_A(\gamma) := & \quad (14) \\ & \int_{\theta_A=0}^{\pi} \int_{\varphi_A=0}^{\frac{\pi}{2}} \mathcal{N}_A(\hat{Q}_A^*, \hat{Q}_B^*, \theta_A, \varphi_A, \gamma) d\theta_A d\varphi_A - \\ & \int_{\theta_A=0}^{\pi} \int_{\varphi_A=0}^{\frac{\pi}{2}} \mathcal{N}_A(\hat{D}_A^*, \hat{D}_B^*, \theta_A, \varphi_A, \gamma) d\theta_A d\varphi_A, \end{aligned}$$

where the functions  $\mathcal{N}_A(\hat{Q}_A^*, \hat{Q}_B^*, \theta_A, \varphi_A, \gamma)$  and  $\mathcal{N}_A(\hat{D}_A^*, \hat{D}_B^*, \theta_A, \varphi_A, \gamma)$  are given by

$$\begin{aligned} \mathcal{N}_A(\hat{U}_A^*, \hat{U}_B^*, \theta_A, \varphi_A, \gamma) = & \quad (15) \\ \mathcal{N}_A(\hat{U}_A^*, \hat{U}_B^*, \hat{U}_A, \gamma) := & \$A(\hat{U}_A^*, \hat{U}_B^*, \gamma) - \$A(\hat{U}_A, \hat{U}_B^*, \gamma) \quad . \end{aligned}$$

A rather lengthy calculation gives the following analytic result for the function  $\mathcal{N}(\gamma) := \mathcal{N}_A(\gamma) = \mathcal{N}_B(\gamma)$  of a two-player quantum game with a prisoner's dilemma payoff matrix:

$$\begin{aligned} \mathcal{N}(\gamma) = & \frac{\pi^2}{16} [(1 + 3 \cos(2\gamma))(a - b) + \\ & (5 - \cos(2\gamma))(c - d)] \quad . \quad (16) \end{aligned}$$

For a separable game equation 16 reduces to  $\mathcal{N}(\gamma = 0) = \frac{\pi^2}{16}(a - b + c - d)$ . Restricting to games with a prisoner's dilemma game structure (see conditions 2) leads always to a negative value of  $\mathcal{N}(\gamma = 0)$ , which means that the classical limit of a quantum prisoner's dilemma game always yield to the classical Nash equilibrium of defection (strategy  $D$ ). In the next but one section we will show the functions  $\mathcal{N}(\gamma)$  for all games used in [1] (Fig. 3) and [2] (Fig. 4).

An integration of  $\mathcal{N}(\gamma)$  from  $\gamma = 0$  to  $\gamma = \frac{\pi}{2}$  leads to a function  $\mathcal{N}$ , that depends solely on the payoff parameters (a, b, c, d).

$$\mathcal{N} := \int_{\gamma=0}^{\frac{\pi}{2}} \mathcal{N}(\gamma) d\gamma = \frac{\pi^3}{32} [a - b + 5(c - d)] \quad . \quad (17)$$

In the following,  $\mathcal{N}$  will be used as the main cooperation indicator.[33] It is easy to show that the null of  $\mathcal{N}(\gamma)$  is given by a specific threshold value  $\gamma_*$  of the entanglement:

$$\begin{aligned} \gamma_* := & \left\{ \gamma \in [0, \frac{\pi}{2}] : \mathcal{N}(\gamma) = 0 \right\} \\ \gamma_* = & \frac{\pi}{2} - \frac{1}{2} \arccos \left( \frac{a - b + 5(c - d)}{3(a - b) - c + d} \right) \quad . \quad (18) \end{aligned}$$

In addition to  $\mathcal{N}$  and  $\gamma_*$ , two other cooperation indicators are defined:  $\gamma_1$  is defined as the entanglement barrier, for which the classical Nash equilibrium  $|DD\rangle$  dissolves, and  $\gamma_2$  is defined as the barrier where the new quantum Nash equilibrium  $|QQ\rangle$  appears (for a detailed discussion of  $\gamma_1$  and  $\gamma_2$  see [4, 7]).

To visualize the quantum game theoretical foundations of our Ansatz and to illustrate the function  $\mathcal{N}_A$  (see equation 14) the two integration components  $\mathcal{N}_A(\hat{Q}_A^*, \hat{Q}_B^*, \theta_A, \varphi_A, \gamma)$  and  $\mathcal{N}_A(\hat{D}_A^*, \hat{D}_B^*, \theta_A, \varphi_A, \gamma)$  are displayed in Fig. 1 for six different  $\gamma$ -values. The grey surface depicts  $\mathcal{N}_A(\hat{D}_A^*, \hat{D}_B^*, \theta_A, \varphi_A, \gamma)$  as a function of the decision angles  $\theta_A$  and  $\varphi_A$ , whereas the wired white surface specifies  $\mathcal{N}_A(\hat{Q}_A^*, \hat{Q}_B^*, \theta_A, \varphi_A, \gamma)$ . In all of the presented illustrations the payoff structure of game 1 of [1] was used (a = 70, b = 100, c = 90 and d = 80). The left picture at the top of Fig. 1 illustrates the separable situation, where no entanglement is present ( $\gamma = 0$ ). For all possible decision angles the grey surface lies above zero, which means that the strategy  $\hat{D}$  is a quantum Nash equilibrium (see equation 15 and definition 13). The white surface lies in contrast always below the zero value, which reveals the futility of the quantum strategy  $\hat{Q}$  within

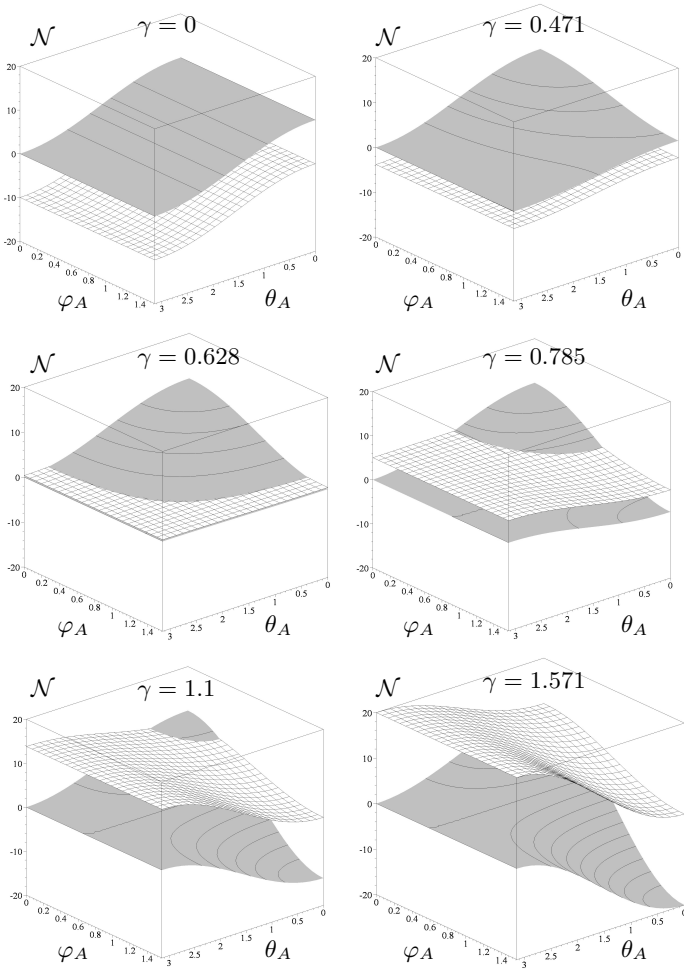


FIG. 1: Visualization of the surfaces  $\mathcal{N}_A(\hat{Q}_A^*, \hat{Q}_B^*, \theta_A, \varphi_A, \gamma)$  (wired white) and  $\mathcal{N}_A(\hat{D}_A^*, \hat{D}_B^*, \theta_A, \varphi_A, \gamma)$  (grey) as a function of the decision angles  $\theta_A$  and  $\varphi_A$  for six different values of entanglement ( $\gamma$ -values). The figures were calculated using the payoff parameters of game 1 of [1] ( $a=70$ ,  $b=100$ ,  $c=90$  and  $d=80$ ).

the separable game. To calculate  $\mathcal{N}_A(\gamma)$ , the whole integration area of the grey surface is subtracted from the integration area of the white surface, which is anyway negative. For  $\gamma = 0$ ,  $\mathcal{N}_A$  becomes therefore highly negative ( $\mathcal{N}_A(\gamma = 0) = -50$ ). The right picture at the top of Fig. 1 shows the resulting surfaces, in the case where the value of entanglement is low ( $\gamma = \frac{3\pi}{20} \approx 0.471$ ). Due to the increase of entanglement both surfaces have converged, but from a qualitative viewpoint the resulting situation has not changed. The grey surface is still above the white surface and in addition, always above zero, which means that defection is still the only Nash equilibrium of the game. The left and right picture in the middle region of Fig. 1 shows the resulting surfaces for a further increase of entanglement (left:  $\gamma = \frac{\pi}{5} \approx 0.628$  and right:  $\gamma = \frac{\pi}{4} \approx 0.785$ ). For  $\gamma = \frac{\pi}{5}$  the white sur-

face lies always above zero, whereas the grey one is for a part of the surface somewhat below zero, which means that the old Nash equilibrium  $\hat{D}$  has disappeared and the new Nash equilibrium  $\hat{Q}$  has become present. The used  $\gamma$ -value ( $\gamma \approx 0.628$ ) lies above the cooperation indicators  $\gamma_1$  and  $\gamma_2$ , which are for this game both equal ( $\gamma_1 = \gamma_2 = 0.615$ ). The integral  $\mathcal{N}_A(\gamma = \frac{\pi}{5})$  is still sparsely negative, whereas the integral  $\mathcal{N}_A(\gamma = \frac{\pi}{4})$  is positive. The left and right pictures in the lower region of Fig. 1 depict the situation where a strong entanglement is present. For the completely entangled game (right picture) the white surface lies always above the grey one and the integral  $\mathcal{N}_A(\gamma = \frac{\pi}{4})$  reaches the largest value.

Figure 1 on the one hand visualizes the structure of game 1 of [1] within a quantum extension of the game and on the other hand it illustrates the integration procedure introduced in equation 14. The shape and the location of the surfaces is important for understanding the properties of a given game and we will present and discuss the other games of [1, 2] in a detailed report [13]. The introduced way of integration when defining the function  $\mathcal{N}_A(\gamma)$  is only one possibility of constructing a cooperation indicator for games with a symmetric payoff matrix. The definition of a more general function  $\mathcal{N}_A(\gamma)$ , which could in addition be used to describe asymmetric games is remaining in employment [13]. Beside the concern of the present article to describe the extent of cooperation in real two-person games the authors think, that the quantum game theoretical method is by all means a valuable tool and new way of understanding the structure of a specific game.

## CLASSICAL VS. QUANTUM COOPERATION INDICATORS

The mathematical description of quantum game theory presented in the previous section is merely a simple one shot quantum game. In contrast, the experiments in [1, 2] are repeated versions of a prisoner's dilemma game. Within such repeated, extensive games the whole strategy sets should be used to describe the game's structure. Within this, primarily examination we neglect such differences by using only the period averaged value of the cooperation percentage  $C_p$  of the experiments [1, 2]. The mathematical formulations of a time dependent quantum game theory describing the dynamics of a population of players is to be working on. In the limit of a separable game such time dependent equations should fade to evolutionary game theoretical concepts and replicator dynamics [14, 15, 16, 17, 18].

The evolution of cooperation in repeated games depends on the payoff parameters of the game and the continuation probability  $\delta$ . [34] Even though the theory of infinitely repeated games has been used to explain cooperation in a variety of environments it does not provide

sharp predictions since there may be a multiplicity of equilibria [2].

In the classical theory of infinitely repeated games the standard lower bound on discount factors ( $\underline{\delta}$ ) below which no player can ever cooperate on an equilibrium path of  $\Gamma(\delta)$  depends simply on the payoff parameters  $b, c$  and  $d$  [10, 19]:

$$\underline{\delta} := \frac{b - c}{b - d} . \quad (19)$$

Cooperation can be achieved by some equilibrium if and only if the continuation probability  $\delta$  is above or equal to the lower bound  $\underline{\delta}$  ( $\delta \geq \underline{\delta}$ ). On the other hand, it is possible to show, that cooperation can be achieved "easily" by a "tit-for-tat" strategy if and only if  $\delta \geq \frac{b-c}{a-d}$  [10].

Blonski et al. have defined a new bound on the discount factors ( $\delta^*$ ), which includes the "sucker's payoff" (parameter "a" of the payoff matrix (see Table I))

$$\delta^* := \frac{b - a - c + d}{b - a} . \quad (20)$$

The authors of [1] show in their article, that this indicator is able to predict the cooperation percentage much better than the standard indicator  $\underline{\delta}$ .

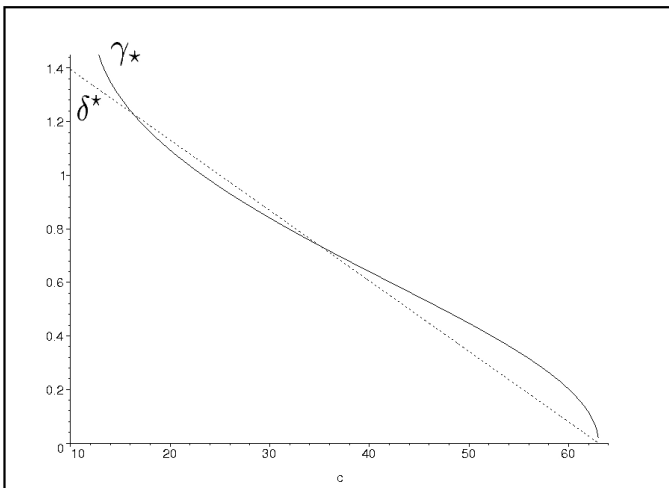


FIG. 2:  $\delta^*$  (dashed line, see [1]) and  $\gamma_*$  (solid curve) as a function of the payoff parameter  $c$ .

It is remarkable, that  $\gamma_*$  and  $\delta^*$  are for a wide range of possible payoff parameters quite similar. Figure 2 illustrates the similarities of the functions  $\gamma_*$  (solid curve) and  $\delta^*$  (dashed line) by varying the parameter  $c$  while keeping the other payoff parameters fixed as in the experimental settings of Dal Bó et. al. [2] ( $a = 12, b = 50, d = 25$ ).

## EXPERIMENTAL VALIDATION

Different variations on the prisoner's dilemma game have been the subject of an enormous experimental in-

terest since the 1950 experiment of Dresher and Flood [10, 20]. Most of the studies have focused on the finitely repeated prisoner's dilemma game [10]. In order to verify the theoretical predictions coming from a quantum game theoretical description it is useful to have data of three or more different payoff parameter settings in one experiment. Unfortunately [21, 22, 23] have only used less than three different payoffs in their studies. Another basic condition is the postulation that an entanglement of strategic choices consists only, if two persons play the game. The outstanding experiments accomplished by Roth and Murnighan [19, 24] had used an experimental setting where a player played against a computer program.[35] Other newer experimental studies have used additional game rules [25] or have analyzed all kinds of asymmetric games in their studies [26].

The experimental designs adopted in the studies [1, 2] are quite similar. Both experiments have used more than two payoff settings and were played by two real persons. Figure 3 shows the function  $\mathcal{N}(\gamma)$  for the six different games used within the experiment [1]. For game 3 and 4 the functions  $\mathcal{N}(\gamma)$  are not distinguishable from each other, because  $\mathcal{N}(\gamma)$  depends only on the difference of the payoff parameters  $a$  and  $b$  (see equation 16).

In the following we will briefly describe the design used in [1]. In each session a group of 20 undergraduate students have participated in the experiment, where they were able to win between 15 to 25 Euro. Ten couples were randomly matched at the beginning of a so called "stage game", whereupon the players could not meet the

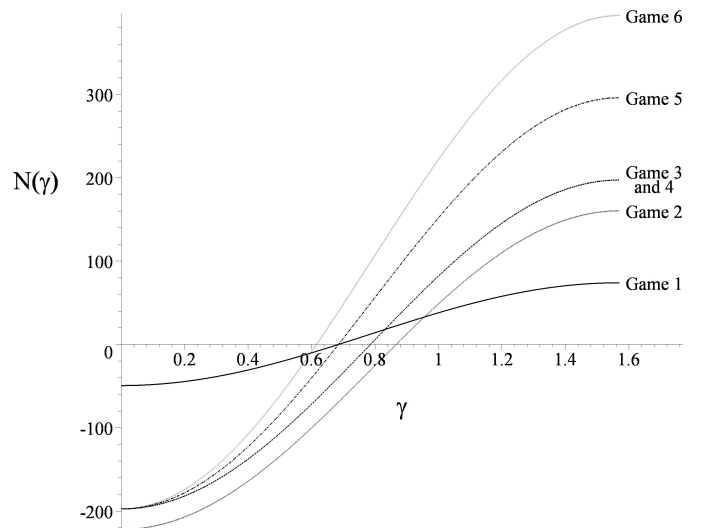


FIG. 3:  $\mathcal{N}(\gamma)$  for the six different games used in the experiment [1]. The quantum cooperation indicator  $\gamma_*$  is the null of the respective functions, whereas  $\mathcal{N}$  is calculated through the integral from  $\gamma = 0$  to  $\gamma = \pi$ .

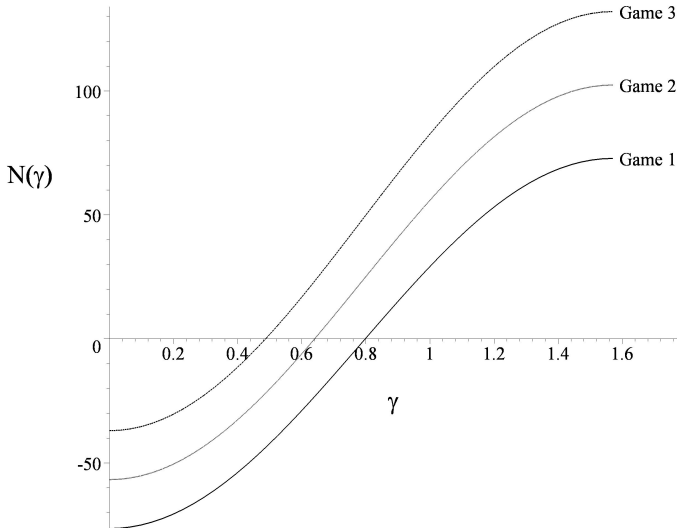


FIG. 4:  $\mathcal{N}(\gamma)$  for the three different games used in the experiment [2].

other one since their decisions were anonymously transmitted by computers. A stage game consisted of a given payoff matrix and a continuation probability  $\delta$ . Six different payoff matrices (see Table II) and three different continuation probabilities ( $\delta = 0.5, 0.75, 0.875$ ) had been specified. During a stage game the continuation probability  $\delta$  and the corresponding opponent did not change. Before every new round the computer picked randomly a probability  $\delta'$  from a uniform distribution ( $\delta' \in [0, 1]$ ) and the game was only continued if  $\delta' \leq \delta$ . Every round consisted of a finite decision phase and an information phase that informed the players about the decision of their opponent and about the achieved payoff. The whole experiment lasted two to three hours.

The design of experiment [2] has only some minor modifications. For instance, the size of the groups of under-

graduate students varied between 12 to 20 subjects, there were only two continuation probabilities ( $\delta = 0.5, 0.75$ ) and three different payoff matrices (see Table II) taken and the achieved payoffs varied between 16 to 43 Dollars. Figure 4 shows the function  $\mathcal{N}(\gamma)$  for the three different games used within the experiment [2].

Quantum theoretical results of the games used in [1, 2] and their experimental data is summarized in Table II and partly visualized in Figure 5. The experimental data is based on the percentage of cooperating persons in all rounds. [36] In the sixth column of Table II the experimental findings of the percentage of cooperating persons ( $C_p$ ) of Blonski et al. [1] and Dal Bó. et al. [2] are denoted, whereas in the seventh column the cooperation rank of the games is quoted. The last rank in experiment [1] for example was found for game 2 ( $C_p = 2.8\%$ ), whereas the lowest cooperation rank was achieved in game 6 ( $C_p = 37.6\%$ ). The next two subsequent columns in Table II present the lower bounds on the discount factors coming from standard ( $\underline{\delta}$ ) and extended ( $\delta^*$ ) classical game theory. The last four columns sum up the specified cooperation indicators calculated with the use of quantum game theory.  $\mathcal{N}$  is considered as the most important indicator. Only if  $\mathcal{N}$  is equal for two games, the indicator  $\gamma_*$  should be used to classify the cooperation rank. In the games 3 and 4 of [1] neither  $\mathcal{N}$  nor  $\gamma_*$  provide distinguishable values. In such a case one can use  $\gamma_1$  and  $\gamma_2$  to classify the cooperation rank, where  $\gamma_1$  is expected to be more important than  $\gamma_2$  because in real two-person games decisions depend firstly on the real strategy choices and only secondly on their imaginary parts. In game 3 the classical Nash equilibrium  $|DD\rangle$  disappears at  $\gamma_1 = 0.685$ , whereas in game 4 it vanishes at  $\gamma_1 = 0.991$ , which means that one expects to have more cooperating persons within game 3.

Figure 5 depicts the percentage of cooperating persons in both experiments as a function of  $\mathcal{N}$ . The diagram clearly shows, that an increase of cooperation comes along with an increase of  $\mathcal{N}$ .

TABLE II: Quantum theoretical results and experimental data of Blonski et.al. [1] and Dal Bó et al. [2].

Experimental data of Blonski et. al. [1]							and quantum theoretical results						
Game No.	a	b	c	d	$C_p$	Rank	$\underline{\delta}$	$\delta^*$	$\gamma_2$	$\gamma_1$	$\gamma_*$	$\mathcal{N}$	
1	70	100	90	80	21.4 %	3	0.5	0.667	0.615	0.615	0.685	19.38	
2	0	100	90	80	2.8 %	6	0.5	0.9	0.322	1.107	0.866	-48.45	
3	30	130	90	70	15.4 %	4	0.667	0.8	0.685	0.685	0.785	0	
4	0	100	90	70	13.4 %	5	0.333	0.8	0.322	0.991	0.785	0	
5	0	120	90	50	37.0 %	2	0.429	0.667	0.524	0.702	0.685	77.52	
6	0	140	90	30	37.6 %	1	0.625	0.786	0.641	0.481	0.615	155.03	
Experimental data of Dal Bó et. al. [2]							and quantum theoretical result						
Game No.	a	b	c	d	$C_p$	Rank	$\underline{\delta}$	$\delta^*$	$\gamma_2$	$\gamma_1$	$\gamma_*$	$\mathcal{N}$	
1	12	50	32	25	7.6 %	3	0.72	0.816	0.759	0.625	0.798	-2.91	
2	12	50	40	25	22.1 %	2	0.4	0.605	0.539	0.625	0.640	35.85	
3	12	50	48	25	28.7 %	1	0.08	0.395	0.231	0.625	0.487	74.61	

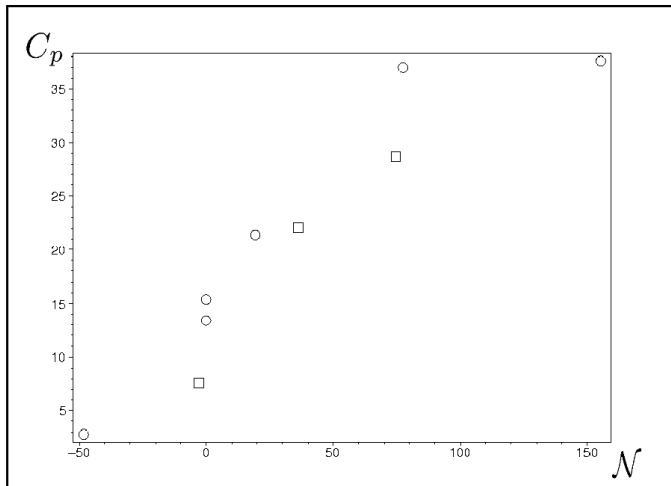


FIG. 5: Percentage of cooperating players ( $C_p$ ) in experiment [1] (circles) and [2] (boxes) as a function of  $\mathcal{N}$ .

It should be mentioned that the comparison of two different experiments is difficult, because besides the fixed payoff parameters and the abruption rate  $\delta$  other experimental details could influence the persons' cooperation behavior. For instance the distribution of the persons' strategic entanglement may depend on cultural characteristics or maybe influenced by the experimental design. The information communicated by the experimenter himself could subliminally or even consciously influence the entanglement distribution of the whole group. Fig. 5 indicates a small difference between the mean of the persons' entanglement in both experiments, because the cooperation percentage in [1] is always somewhat above experiment [2].

An increase (decrease) of  $\delta$  influences the distribution of the players' entanglement, which results in an increase (decrease) of  $C_p$ . The strong correlation between  $\mathcal{N}$  and  $C_p$  for the specific games remains [13].

Our work does not contradict the results of [8], but we presume, that by implementing a specific quantum version of the prisoner's dilemma game, the experimenters have increased the strength of entanglement of the players' strategic decisions (and as a result the cooperation percentage  $C_p$ ).

## SUMMARY

This article shows that a quantum extension of classical game theory is able to describe the experimental findings of two-person prisoner's dilemma games. A classification scheme was introduced to evaluate the cooperation hierarchy of prisoner's dilemma games. Four cooperation indicators were defined to predict the cooperation behavior. This quantum game theoretical approach was compared with predictions based on classical game the-

ory and tested for two experimental settings. To answer our research question, we conclude that compared to cooperation indicators based on classical game theory the defined "quantum" indicators predict the outcome of real person game experiments very good.

## Acknowledgments

We want to thank Peter Ockenfels and Matthias Blonski for helpful discussions. Furthermore we want to thank the referees for several helpful suggestions which had result in an improvement of the original article [27]. This research is supported by grants from the German National Science Foundation (DFG) (Project Scientific Publishing and Alternative Pricing Mechanisms, Grant No. GZ 554922). We gratefully acknowledge the financial support.

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- [32] An introduction into the main concepts of quantum game theory can be found at the online lecture notes (mostly in German) [28, 29, 30].
- [33] The cooperation percentage of a particular experiment should depend on the payoff parameters (a, b, c, d) of the game used in the experiment but will also be dependent on its specifications and given rules. The continuation probability  $\delta$  used in the experiments of [1, 2] for example is such an additional input, which will change the resulting cooperation percentage. Within a quantum description of the experiment we interpret such additional specifications as a shift of the entanglement distribution ( $\nu(\gamma)$ ) of the participators of the experiment. If we want to use  $\mathcal{N}$  as a real cooperation indicator, it should also depend on the entanglement distribution  $\nu(\gamma)$  ( $\mathcal{N} = \int_{\gamma=0}^{\frac{\pi}{2}} \nu(\gamma) \mathcal{N}(\gamma) d\gamma$ ). We have neglected such an inclusion of the entanglement distribution, so that we can use  $\mathcal{N}$  solely within experimental comparisons of equal specifications.
- [34] The present paper solely compares the classical theory of infinitely repeated games with the quantum approach. A comparison with the more general formulations based on negotiation and the axiomatic approach of two-player cooperative games [31] will be addressed in a separate article [13].
- [35] If we take the data from the experiment [24] to test the predictions derived from a quantum game theoretical approach, we do not get a good agreement. We think that the reason of this disagreement is that a player in this experiment played against a computer program and not against another player.
- [36] For comparison reasons we have used the data with  $\delta = 0.75$  in both experiments. Besides the used payoff parameters, the discount factor  $\delta$  is an important property in all of the experiments. As  $\delta$  describes the abrupt probability of the repeated games, it increases the individual entanglement of the persons, which results in a higher percentage of cooperation.