A note on the stability of voting games

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The models of vote in this work are devoted to bill adoption contexts. In such a context, a bill is considered by the community. Assume for instance that the issue is a referendum on whether or not to amend the constitution. The constitution in force (say y) is called the status quo. A projected constitution (say x) is a candidate to the replacement of y. Enforcing the bill consists of adopting the replacement, while rejecting the bill consists of discarding the candidate x. So, there are two options for the collective decisions : bill adopted, or bill rejected.

Simple games, also called yes-no system, introduced by von Neumann (1944) assume that a coalition of voters is either decisive (otherwise called winning, in which case it holds the entire power of decision), or losing (in which case it is absolutely powerless). In such model, there are no provision for abstentions, and abstentions (if any occurs) have effects that are equivalent to votes against the bill.

In the well known relative majority voting, the collective decision enforces the bill if the number of voters who vote for the bill is greater than the number of those who vote against. Social decision systems (Sds), owed to Rubinstein (1980), are a generalization of the relative majority voting. In addition to the two voting options (for or against), a voter can also abstain. The casting of all individual votes thus realizes a partition of the set of all voters into three subsets. Those who abstain, those who vote for and those who vote against the bill. Decisiveness is therefore described using couples of disjoints coalitions.

We study the core of Sds, with emphasis on the characterization of its non-emptiness for every preference profile of voters. Such a characterization for simple games was obtained by Nakamura (1979). Herein, we obtain a necessary and sufficient condition for which the core of an Sds is non-empty regardless of the profile of individual preferences, and we show that our result is a generalization of the Nakamura's theorem. As a corollary, we show that the relative majority voting is not stable, provided that there are at least three voters.