

ILLC MASTERCLASS LOGICA 2017

Logica en Wiskunde
9 March 2017, 11:45–14:15
Universiteit van Amsterdam

1. Fix some $n \in \mathbb{N}$. In the example of the three boxes and the red balls, find a labelling of the red balls that allows you to argue that after infinitely many steps, there are exactly n red balls in the second box.
2. In the example of Hilbert's hotel, imagine that just after the bus with infinitely many new guests, a ferry arrives containing infinitely many buses each with infinitely many new guests. How will the receptionist deal with this situation? In which room will the first new guest arriving that evening be after the re-arrangement of the guests for the ferry passengers? What if after that, a fleet of infinitely many ferries each containing infinitely many buses with infinitely many passengers arrives?
3. We claimed that if $X_1 := \{1, \dots, n\}$ and $X_2 := \{1, \dots, m\}$ with their natural order, then the order sum of $(X_1, <)$ and $(X_2, <)$ is the set $\{1, \dots, n + m\}$ with its natural order. Make that claim precise using the notion of isomorphism of ordered structures and prove it.
4. We identified orders $<_1$ and $<_\infty$ on \mathbb{N} that correspond to the arrival of one extra guest and the arrival of a bus with infinitely many guests at Hilbert's hotel. Find subsets X_1 and X_∞ of \mathbb{Q} , the rational numbers, such that $(\mathbb{N}, <_1) \cong (X_1, <)$ and $(\mathbb{N}, <_\infty) \cong (X_\infty, <)$.

Find the appropriate orders on \mathbb{N} for the examples of the ferry of buses and the fleet of ferries from question 2. and find corresponding subsets of \mathbb{Q} .

Can this be generalised to arbitrary wellorders defined on \mathbb{N} ?