TITLE: Closure and the Growth of Error

One reason to care about whether knowledge is closed under known implication is that if it is not then valid deduction allows the growth of potential error. But then whether closure failure should be worrying depends on how fast error grows with every step of deduction, which is a quantitative question. If knowledge is probabilistic tracking then we can express this quantitative question, and derive upper bounds on the growth of error. I show that the growth is slow enough to make sense of the possibility of learning things indirectly by inference, and that the quantitative approach sheds new light on some standard examples of closure failure .