Towards a Logical Account of Binding Theory

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Logical Grammars With Labels
- Some characteristics of \textit{LGL}
- Logical rules

\textit{LGL} & Binding theory
- Binding Theory
- Treatment of reflexive binding in \textit{LGL}
- Treatment of non-reflexive pronouns in \textit{LGL}

Conclusion
Some general characteristics of \textit{LGL}

- Undirected system like \textbf{ACG} [Pdg01] and \textbf{\(\lambda\)-Grammars} [Muskens03]
- \textbf{Abstract level}: syntactic dependencies \(\Rightarrow\) a fragment of linear logic (2 connectives \(-\rightarrow, !\))
- \textbf{Concrete level}: \textit{phonetics} and \textit{semantics} \(\Rightarrow\) \(\lambda\)-terms combination (Curry-Howard homomorphism)

\[
\begin{array}{|c|}
\hline
\text{d}_{\text{acc}} -\rightarrow \text{d}_{\text{nom}} -\rightarrow c \\
\hline
\text{\(\lambda\)x. \(\lambda\)y. } \text{\(y\) \textit{\(\bullet\) reads} \textit{\(\bullet\) x}} \\
\text{\(\lambda\)x. \(\lambda\)y. } \text{\textit{Read}(y, x)} \\
\hline
\end{array}
\]
Some specific characteristics of $LGL$

- Hypothetical reasoning technique is controlled
- The freely accessible logical axiom rule is excluded
- Available axioms (controlled hypotheses) are explicitly given by the lexicon

\[ \lambda x. f(x) : H \rightarrow A \]

\[ f(x) : A \]

\[ [x : H] \]
Some specific characteristics of \textit{LGL}:

- Hypothetical reasoning technique is \textit{controlled}.
- The freely accessible logical axiom rule is excluded.
- Available axioms (\textit{controlled hypotheses}) are explicitly given by the lexicon.

\begin{align*}
\lambda x. f(x) : H \rightarrow o A \\
f(x) : A \\
[x : H]
\end{align*}

\begin{align*}
g : (H \rightarrow o A) \rightarrow o B \\
\Rightarrow [x : H]
\end{align*}

\textit{Linked entry}
Logical Rule 1: Modus-Ponens

\[ f : A \rightarrow B \]

\[ a : A \]
Logical Rule 1: Modus-Ponens

\[ f(a) : B \]

\[
\begin{align*}
  f : A &\rightarrow B \\
  a : A &
\end{align*}
\]
Modus-Ponens (Example)

\[
d_{\text{nom}} \leadsto c
\]

\[
\lambda y. y \cdot \text{reads} \cdot \text{Amok}
\]

\[
d_{\text{acc}} \leadsto d_{\text{nom}} \leadsto c
\]

\[
\lambda x. \lambda y. y \cdot \text{reads} \cdot x \quad \text{Amok}
\]
Modus-Ponens (Example)

\[
c \quad \text{John reads Amok}
\]

\[
d_{\text{nom}} \circ c \quad d_{\text{nom}}
\]

\[
\lambda y. \ y \text{ reads Amok} \quad \text{John}
\]

\[
d_{\text{acc}} \circ d_{\text{nom}} \circ c \quad d_{\text{acc}}
\]

\[
\lambda x. \ \lambda y. \ y \text{ reads } x \quad \text{Amok}
\]
Logical Rule 2: Controlled Hypothetical Reasoning

Using a linear linked entry

\[ g(\lambda x. f(x)) : B \]

\[ g : (H \rightarrow o A) \rightarrow o B \]

\[ \Rightarrow \ [x : H] \]

\[ \lambda x. f(x) : H \rightarrow o A \]

\[ f(x) : A \]

\[ [x : H] \]
Logical Rule 2: Controlled Hypothetical Reasoning
Using a non-linear linked entry

$g(\lambda x. f(x_1 := x, \ldots, x_k := x)) : B$

$g: (!H \rightarrow oA) \rightarrow oB$

$\lambda x. f(x_1 := x, \ldots, x_k := x): !H \rightarrow oA$

$\Rightarrow [x_1 : H], \ldots, [x_k : H]$

$f(x_1, \ldots, x_k) : A$

$[x_1 : H]$

$[x_i : H]$

$[x_k : H]$
Controlled Hypothetical Reasoning (Example)

\[ \lambda m. m \cdot \text{which} \cdot \text{John} \cdot \text{reads} \cdot \varepsilon \]

\[ (d_{acc} \cdot c) \cdot n \cdot n \]

\[ \lambda P \lambda m. m \cdot \text{which} \cdot P(\varepsilon) \ [e_i] \]

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LGL & BT
Principles A & B

- Anaphora should be bound in their local domain
- Non-reflexive pronouns must not be bound within their local domain

Examples

- John$_i$ likes himself$_i$.
- *John$_i$ thinks Bob likes himself$_i$.
- John$_i$ thinks he$_i$ is smart.
- *John$_i$ likes him$_i$. 
Logical Treatment of reflexive binding

Object/Subject reflexivization (‘himself’)

- **Syntax**: a functor which combines with a transitive verb and returns an intransitive verb.
- **Semantics**: a non-linear term, i.e., $\lambda P. \lambda x. P(x, x)$

Problems with previous systems

- Free access to hypothetical reasoning: both ‘likes’ and ‘thinks’ have the same type.
- Violation of locality constraint.
- Proposed solutions: enhancing the core logic with new connectives (e.g., control operator [Morrill90]).
himself vs ziji

**himself**

- Using a free lexical entry (to block recourse to hypothetical reasoning).
- ‘himself’ can only combine with lexical arguments of type $d_{acc} \rightarrow d_{nom} \rightarrow c$ (e.g., ‘likes’).
- Compound expressions (e.g., ‘thinks Bob likes’) cannot be considered as potential arguments.

**ziji (long-distant anaphora)**

Zhangsan$_k$ renwei Lisi$_j$ zhidaowangwu$_i$ xihuan ziji$_{i/j/k}$

Zhangsan renwei Lisi knows Wangwu likes self

‘Zhangsan thinks Lisi knows that Wangwu likes himself’

- Using a linked entry associated to a controlled hypothesis [x: $d_{acc}$].
**himself vs ziji**

- $V_1 = d_{nom} \rightarrow c$ (intransitive verb type).
- $V_2 = d_{acc} \rightarrow V_1$ (transitive verb type).

<table>
<thead>
<tr>
<th>himself</th>
<th>ziji</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>$V_1$</td>
</tr>
<tr>
<td>$\lambda x. x \cdot \text{likes} \cdot \text{himself}$</td>
<td>$\lambda y. y \cdot \text{zhidao} \cdot \text{Wangwu} \cdot \text{xihuan} \cdot \text{ziji}$</td>
</tr>
<tr>
<td>$\lambda x. \text{Like}(x, x)$</td>
<td>$\lambda y. \text{Know}(y, \text{Like}(\text{Wangwu}, y))$</td>
</tr>
</tbody>
</table>

\[ V_2 \dashv V_1 \]

\[ \lambda P. \lambda x. P(\text{himself}, x) \quad \lambda P. \lambda x. P(\text{ziji}, y) \]

\[ \lambda P. \lambda x. P(x, x) \quad \lambda x. \lambda y. \text{likes} \cdot x \]

\[ \lambda x. \lambda y. \text{Like}(y, x) \quad \lambda x. \lambda y. \text{Like}(y, x) \]

\[ ([x_\phi, x_\lambda])^I: d_{acc} \]
Non-reflexive pronouns

Kayne proposal [Kayne02]

\[ \text{thinks [John, he] is smart} \rightarrow \text{John}_i \text{ thinks [} t_i, \text{ he} \text{] is smart} \]

\[ (\ast) \ [\text{John, he}] \text{ thinks is smart} \rightarrow [t_i, \text{ he}] \text{ thinks John}_i \text{ is smart} \]

\[ \text{thinks John likes [Bob, him]} \rightarrow \text{Bob}_i \text{ thinks [} t_i [\text{John likes [} t_i, \text{ him}]] \text{]} \]

\[ (\ast) \ \text{likes [John, him]} \rightarrow [t_i [\text{John}_i \text{ likes [} t_i, \text{ him}]]] \]
Modeling the doubling constituent [John, him]

$$e_3 : \left( \lambda P_\phi \cdot \text{John} \bullet P_\phi(\epsilon) \right) \cdot \lambda P_\lambda \cdot P_\lambda(\text{John}) : (!d \circ c) \circ c \rightarrow \neg \neg$$

$$\begin{align*}
(H_1 & : [(x_{\lambda 1}, x_{\phi 1}) : d_{\text{nom}}], \\
H_2 & : [(\lambda y_\phi\cdot y_\phi, \lambda y_\lambda\cdot y_\lambda) : c \circ c] \\
H_3 & : [(\text{him}, x_{\lambda 2}) : d_{\text{acc}}])
\end{align*}$$

- [H_1]: occupies the antecedent position.
- [H_2]: intermediary position which delimits the local domain
- [H_3]: occupies the position of the pronom him.
- A necessary condition: controlled hypotheses should be introduced in that order (H_3, H_2, H_1).
H₂ hypothesis is introduced after H₁ ⇒ the binding between ‘John’ and ‘him’ is forbidden.
Bob\(_i\) thinks John likes him\(_i\)

- \(H_3\) is the *first* controlled hypothesis to be used.

\[
\begin{align*}
\lambda x. \lambda y. \text{Like}(y, x) \\
\lambda y. \text{Like}(y, x) \\
\lambda y. \text{Like}(y, x) \\
\lambda y. \text{Like}(y, x) \\
\end{align*}
\]
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Bob_i thinks John likes him_i

- \( H_2 \) hypothesis is introduced before \( H_1 \) \( \Rightarrow \) the antecedent position is outside the local domain of ‘him’.

\[
\begin{align*}
\text{Think}(x_{\lambda 1}, \text{Like}(\text{John}, x_{\lambda 3})) \\
\text{Think}(y_{\lambda}, \text{Like}(\text{John}, x_{\lambda 3})) \\
\text{Think}(y_{\lambda}, P_{\lambda}) \\
\text{Like}(\text{John}, x_{\lambda 3})
\end{align*}
\]
Bob\textsubscript{i} thinks John likes him\textsubscript{i}

- **Contraction & simultaneous abstraction** of controlled hypotheses ⇒ binding the pronoun ‘him’ with its antecedent ‘Bob’.

\[
\begin{array}{c}
\text{Think}(\text{Bob}, \text{Like}(\text{John}, \text{Bob})) \\
\end{array}
\]

\[
\begin{array}{c}
\lambda P_{\phi}. \text{Bob} \cdot P_{\phi}(\varepsilon) \\
\lambda P_\lambda. P_\lambda(\text{Bob}) \\
\end{array}
\]

\[
\begin{array}{c}
\text{Think}(x_{\lambda 1}, \text{Like}(\text{John}, x_{\lambda 3})) \\
\end{array}
\]
Summary

- Locality constraints (Principle A): controlling hypothetical reasoning in \textit{LGL}.
- The antecedent-pronoun relation: using linked entries (binding $\leftrightarrow$ contraction + simultaneous abstraction of controlled hypotheses).
- Principle B: using a hypothesis to delimit the local domain + constraints on the order of introduction of controlled hypotheses.

Outlook

- Interaction between anaphora and other linguistic phenomena (e.g., VP-ellipsis, ‘John loves his mother and Bob does too’).
- Uniform modeling of binding theory (logical formalization of Chomsky’s phase theory [Chom01]).


