Computational Epistemology for Quantifiers

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VIIth Tbilisi Symposium
October 5, 2007
1 PROBLEMS

2 QUANTIFIERS
   - Quantifiers of type \( \langle 1 \rangle \)
   - Quantifiers of type \( \langle 1, 1 \rangle \)

3 COMPUTATIONAL EPISTEMOLOGY

4 IDENTIFIABILITY

5 GENERAL QUESTION
Plan

1 Problems

2 Quantifiers
   - Quantifiers of type \langle 1 \rangle
   - Quantifiers of type \langle 1, 1 \rangle

3 Computational Epistemology

4 Identifiability

5 General question
PROBLEMS

- Epistemological properties of quantifiers.
- Their influence on NL comprehension.
- Linking them to learnability features.
- Compare notions of decidability and identifiability.
1. **Problems**

2. **Quantifiers**
   - Quantifiers of type $\langle 1 \rangle$
   - Quantifiers of type $\langle 1, 1 \rangle$

3. **Computational Epistemology**

4. **Identifiability**

5. **General Question**
A monadic generalized quantifier of type \((1, \ldots, 1)\) is a class \(Q\) of structures of the form \(M = (M, A_1, \ldots, A_n)\), where \(A_i\) is a subset of \(M\). Additionally, \(Q\) is closed under isomorphism.
**Q OF TYPE** \(\langle 1\rangle\)

**Monotonicity**

**Definition**

\(Q_M\) is \(\text{MON}^{\uparrow}\) iff: if \(A \subseteq A' \subseteq M\), then \(Q_M(A)\) implies \(Q_M(A')\).

**Definition**

\(Q_M\) is \(\text{MON}^{\downarrow}\) iff: if \(A' \subseteq A \subseteq M\), then \(Q_M(A)\) implies \(Q_M(A')\).
Q OF TYPE $\langle 1 \rangle$

**Extendability**

**Definition**

Q of type $\langle 1 \rangle$ satisfies $\text{EXT}$ iff for all models $M$ and $M'$: $A \subseteq M \subseteq M'$ implies $Q_M(A) \iff Q_{M'}(A)$. 
Restriction to CE quantifiers.

**Definition**

Let $Q$ be of type $\langle 1, 1 \rangle$. Then for all $M, M'$, all $A, B \subseteq M$, and $A', B' \subseteq M'$:

(ISOM) If $(M, A, B) \cong (M', A', B')$, then $Q_M(A, B) \iff Q_{M'}(A', B')$.

(CONS) $Q_M(A, B) \iff Q_M(A, A \cap B)$.

(EXT) If $M \subseteq M'$, then $Q_M(A, B) \iff Q_{M'}(A, B)$. 
CE QUANTIFIERS
(ISOM) IF \((M, A, B) \cong (M', A', B')\), THEN \(Q_M(A, B) \iff Q_{M'}(A', B')\)
CE QUANTIFIERS - EXT

(EXT) If $M \subseteq M'$, then $Q_M(A, B) \iff Q_{M'}(A, B)$
CE QUANTIFIERS - CONS

(CONS) $Q_M(A, B) \iff Q_M(A, A \cap B)$
The scope we are interested in for both $\langle 1 \rangle$ and $\langle 1, 1 \rangle$ cases.
Q OF TYPE $\langle 1, 1 \rangle$

Monotonicity

Definition

Q of type $\langle 1, 1 \rangle$ is $\text{MON}^\uparrow$ iff:
If $A \subseteq M$ and $B \subseteq B' \subseteq M$, then $Q^M(A, B) \Rightarrow Q^M(A, B')$. 
Q OF TYPE $\langle 1, 1 \rangle$

PERSISTENCE

**DEFINITION**

Q of type $\langle 1, 1 \rangle$ is PER iff:
If $A \subseteq A' \subseteq M$ and $B \subseteq M$, then $Q_M(A, B) \Rightarrow Q_M(A', B)$.
### Examples

<table>
<thead>
<tr>
<th>Determiner</th>
<th>MON ↑</th>
<th>EXT (for $\langle 1 \rangle$)</th>
<th>PER (for $\langle 1, 1 \rangle$)</th>
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**Table:** Quantifiers and their properties
Monotonicity & Linguistics

- Does monotonicity influence NL comprehension?
- Does monotonicity influence NL learning?
- Monotonicity and inference patterns (B. Geurts).
- Proposal: focus on persistence.
PLAN

1 PROBLEMS

2 QUANTIFIERS
   • Quantifiers of type ⟨1⟩
   • Quantifiers of type ⟨1, 1⟩

3 COMPUTATIONAL EPISTEMOLOGY

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5 GENERAL QUESTION
Logic of reliable inquiry - Kevin Kelly

- Inspired by learning theory.
- Similar framework.
- Verification/falsification in computational setting.
Logic of reliable inquiry - Kevin Kelly

\( \varepsilon \) — infinite string of data;
\( \varepsilon|n \) — finite initial segment of \( \varepsilon \) through the position \( n - 1 \);
\( h \) — hypothesis;
\( C \) — correctness relation;
\( C(\varepsilon, h) \) iff \( h \) is correct w.r.t. \( \varepsilon \);
\( \alpha \) — an assessment method;
\( \text{OUT} \) conjectures 1, 0, !.
Certainty in Reliable Inquiry

**Definition**

$\alpha$ produces $b$ with certainty on $(h, \varepsilon)$ iff there is an $n$ s.t.:

1. $\alpha(h, \varepsilon|n) = !$, and
2. $\alpha(h, \varepsilon|n+1) = b$, and
3. for each $m < n$, $\alpha(h, \varepsilon|m) \neq !$. 

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Computational Epistemology for Quantifiers
**Certainty in Reliable Inquiry**

**Definition**

\( \alpha \) verifies \( h \) with certainty on \( \varepsilon \) (with respect to \( C \)) iff \( \alpha \) produces 1 with certainty on \( (h, \varepsilon) \) \( \iff C(\varepsilon, h) \).

**Definition**

\( \alpha \) refutes \( h \) with certainty on \( \varepsilon \) (with respect to \( C \)) iff \( \alpha \) produces 0 with certainty on \( (h, \varepsilon) \) \( \iff \neg C(\varepsilon, h) \).

**Definition**

Decidability with certainty is simply verifiability and refutability with certainty at the same time.
EXAMPLES

- At least six bikes are broken. - Verifiable with certainty
- An even number of bikes is broken. - Verifiable in the limit
EPISTEMOLOGICAL PROPERTIES OF Q o.t. \langle 1 \rangle

- 1–1 enumeration of elements of the universe.
- Assignment of \( \chi_A \) to each of them.
- Infinite sequence of 0s and 1s.
- In each step checking if finite initial segment satisfies a hypothesis (quantifier sentence).
PROPOSITION

Let $Q$ be FO quantifier of type $\langle 1 \rangle$.
$Q$ is MON $\uparrow$ and EXT iff it is verifiable with certainty.

PROPOSITION

Let $Q$ be FO quantifier of type $\langle 1 \rangle$.
$\neg Q$ is verifiable with certainty iff $Q$ is falsifiable with certainty.
**Proposition**

Let $Q$ be FO CE quantifier of type $\langle 1, 1 \rangle$. $Q$ is persistent iff it is verifiable with certainty.

**Proposition**

Let $Q$ be FO CE quantifier of type $\langle 1, 1 \rangle$. $\neg Q$ is falsifiable with certainty iff $Q$ is verifiable with certainty.
### Examples

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3. Computational Epistemology
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5. General question
Class of objects is chosen (e.g. class of grammars).

Player 1 picks out one object from the class (e.g. \( G \)).

Player 1 generates positive instances of this object, repetitions allowed (e.g. words from a language of \( G \)).

Player 2 knows about the class, but he does not know which object is chosen.

Player 2 has to guess which object Player 1 has in mind.
Learning the semantics of natural language
Identifiability from text in use

- Class of quantifiers is chosen.
- Player 1 picks one of them (Q)
- Player 2 is presented finite worlds in which Q is true.
- Player 2 has to identify Q.
Assuming CE, we can represent all relevant models in the form of number triangle.

\[(0,0)\]
\[(1,0) \quad (0,1)\]
\[(2,0) \quad (1,1) \quad (0,2)\]
\[(3,0) \quad (2,1) \quad (1,2) \quad (0,3)\]
\[(4,0) \quad (3,1) \quad (2,2) \quad (1,3) \quad (0,4)\]
\[\ldots \quad \ldots\]
Number triangle representation

- Graphic representation of a class of CE quantifiers.
- In particular: PER.

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NUMBER TRIANGLE REPRESENTATION

- Graphic representation of a class of CE quantifiers.
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TIEDE’S RESULT

**Theorem**

The class of FO PER Q is identifiable from text.
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General Question
Relation between Ver/Fal hierarchy and identifiability

Decidability
- Gradual Ver
- Gradual Fal
- Gradual Dec
- Limit Ver
- Limit Fal
- Limit Dec
- Certain Ver
- Certain Fal
- Certain Dec

Identifiability
- Gradual Id from informant
- Gradual Id from text
- Limit Id from informant
- Limit Id from text
- Certain Id from informant
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Computational Epistemology for Quantifiers
CONCLUSIONS AND FUTURE WORK

- Epistemological role of monotonicity - additional explanation.
- Verification less difficult than falsification?
- Check connections between persistence and comprehension.
- Investigate relationship between identifiability and decidability: learning of NL semantics; new conditions of identifiability.
THANK YOU!