A Logic for Assertion Networks

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A Logic for Assertion Networks

1 Background
   • Assertion network

2 Logic for Assertion network
   • Graph operations
   • A logical language

3 Conclusions
   • Final notes
## A real life situation

You and your colleague share an office without windows. You are talking on the phone to your friend who is sitting in a street café. You want to know whether the sun is shining outside or not.

<table>
<thead>
<tr>
<th>Friend</th>
<th>Colleague</th>
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<tbody>
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<td>&quot;Everything your colleague says is false; the sun is shining!&quot;</td>
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What to believe?
A real life situation

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Friend: “Everything your colleague says is false; the sun is shining!”
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What to believe?
Directed labelled graphs
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- Agents and facts represented by vertices \((V)\).
Directed labelled graphs

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- Agents’ opinions represented by labelled edges \((l : E \rightarrow \{+, -\})\).
Directly labelled graphs

- Agents and facts represented by vertices \((V)\).
- Agents’ opinions represented by labelled edges \((l : E \rightarrow \{+, -\})\).
- Directed labelled graph (DLG) \(G = (V, E, l)\).
Representing the situation

- $A_c$ - your colleague
- $A_f$ - your friend
- $S$ - sun is shining
Representing the situation

- $A_c$ - your colleague
- $A_f$ - your friend
- $S$ - sun is shining

\[
\begin{array}{c}
A_c \quad \rightarrow \\
\quad \quad \quad \rightarrow \\
A_f \quad \leftarrow \quad \leftarrow \\
\quad \quad \quad \leftarrow \\
S
\end{array}
\]
What to believe?

Definition (Assertion network semantics (AN))

Given $G = (V, E, I)$:
What to believe?

**Definition (Assertion network semantics (AN))**

Given $G = (V, E, l)$:

- Observer’s initial degree of belief (initial hypothesis):

$$H : (V \cup E) \rightarrow (\mathbb{Q} \cap [-1, 1])$$
What to believe?

**Definition (Assertion network semantics (AN))**

Given $G = (V, E, I)$:
- Observer’s initial degree of belief (initial hypothesis):
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- New degree of belief from previous ones:
  $$H_i = \Psi^i(H)$$
What to believe?

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Given $G = (V, E, I)$:

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$$H : (V \cup E) \rightarrow (\mathbb{Q} \cap [-1, 1])$$

- New degree of belief from previous ones:

$$H_i = \Psi^i(H)$$

- $\lambda_s$ stable value of $s \in (V \cup E)$ if $\lim_{i \to \infty} H_i(s) = \lambda_s$

$$\text{St}_H(G, s) = \lambda_s$$
A particular $\psi$

Focus on belief changes in opinions (edges) and facts (terminal vertices).

non-terminal vertices  terminal vertices  edges
**Toolkit**

A Logic for Assertion Networks
**A Logic for Assertion Networks**

**Toolkit**

![Diagram of an assertion network with vertices and edges, along with a table of values for each vertex and edge.]
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Toolkit
A Logic for Assertion Networks
This work

Goal

**Goal:**

*to define a logic to reason about assertion networks*
This work

Restrictions

Some restrictions for this initial work:
This work

Restrictions

Some restrictions for this initial work:

- Finite graphs.
This work

Restrictions

Some restrictions for this initial work:

- Finite graphs.
- Initial opinions just for facts ($s \notin T \Rightarrow H(s) = 0$).
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**Intuitive idea**

- New agents with opinion about existing agents.
- A distinguished non-terminal node (an already considered agent).
- New nodes and edges representing new agents and their opinions.
Intuitive idea

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Negation, conjunction and disjunction

\[ G_1 = (G_1, v_1) \text{ and } G_2 = (G_2, v_2) \text{ two pointed DLG:} \]
Negation, conjunction and disjunction

\[ \mathcal{G}_1 = (G_1, v_1) \text{ and } \mathcal{G}_2 = (G_2, v_2) \text{ two pointed DLG:} \]

\[ \neg v_1 \]

\[ \neg v_1 \downarrow \]

\[ \neg \mathcal{G}_1 \]

\[ \ominus \mathcal{G}_1 \]
Graph operations

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Negation, conjunction and disjunction

\[ G_1 = (G_1, v_1) \text{ and } G_2 = (G_2, v_2) \] two pointed DLG:

\[ \neg v_1 \quad \neg v_1 \quad v_1 \land v_2 \quad v_1 \lor v_2 \quad v_1 \land v_2 \quad v_1 \lor v_2 \]
Examples

$G_1$  

$G_2$  

$G_3$  

A Logic for Assertion Networks
Examples

$G_1$

$G_2$

$G_3$

$\Theta G_2$

$G_1 \odot G_3$
Analyzing simple graphs

\[ G_1 = v_1 \xrightarrow{+} w_1 \quad \text{and} \quad G_2 = v_2 \xrightarrow{+} w_2. \]
Analyzing simple graphs

\[ G_1 = v_1 \rightarrow^+ w_1 \quad \text{and} \quad G_2 = v_2 \rightarrow^+ w_2. \]

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Some properties for simple graphs

\[ G_1 = \overset{+}{v_1} \rightarrow w_1 \quad \text{and} \quad G_2 = \overset{+}{v_2} \rightarrow w_2. \]
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- Duality for \( \ominus \):
  \[ \text{St}_H(G_1) = \text{St}_H(\ominus \ominus G_1) \]
Graph operations

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  \[ \text{St}_H(G_1) = \text{St}_H(G_1 \odot G_1) \quad \text{St}_H(G_1) = \text{St}_H(G_1 \oplus G_1) \]
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- **Commutativity for \( \odot \) and \( \oplus \):**
  \[ \text{St}_H(G_1 \odot G_2) = \text{St}_H(G_2 \odot G_1) \quad \text{St}_H(G_1 \oplus G_2) = \text{St}_H(G_2 \oplus G_1) \]
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Syntax

Definition (Syntax)

Given a set $\Phi$ of atomic propositions (agents), $L_{\text{ANT}}$ is the smallest set of agent terms containing $\Phi$ and closed under $\neg$, $\land$, $\lor$. 
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Semantics

\( \mathcal{G} \) the class of all pointed DLGs.
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**Definition (Semantic model)**

A pair \( \langle K, H \rangle \) where

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A pair \( \langle K, H \rangle \) where

- \( K : \Phi \to \mathcal{G} \) is an \( \mathcal{L}_{DLG} \)-assignment, extended for \( \mathcal{L}_{ANT} \) as
  - \( K(\varphi \land \psi) := K(\varphi) \odot K(\psi) \)
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- \( H \) is an initial hypothesis.

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- $H$ is an initial hypothesis.

**Definition (Semantics)**

$\langle K, H \rangle \models \varphi$ iff $\text{St}_H(K(\varphi)) > 0$
Semantics

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- \( H \) is an initial hypothesis.

**Definition (Semantics)**

\[ \langle K, H \rangle \models \varphi \quad \text{iff} \quad \text{St}_H(K(\varphi)) > 0 \]

\[ \langle K, H \rangle \models_{\geq} \varphi \quad \text{iff} \quad \text{St}_H(K(\varphi)) \geq 0 \]
Semantical entailment relation

Intuitive idea:
Semantical entailment relation

Intuitive idea:
- How beliefs over one agent influences the beliefs in other.
- Same general situation (discarding disjoint graphs and unrelated initial hypothesis).
Semantical entailment relation

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\( \varphi, \psi \) agent terms, with one of them a subterm of the other.
Semantical entailment relation

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Definition (Entailment relation)

$\varphi, \psi$ agent terms, with one of them a subterm of the other.

$\varphi \models_{\langle K, H \rangle} \psi$ if $\langle K, H \rangle \models_{\geq} \psi$ whenever $\langle K, H \rangle \models_{>} \varphi$
Semantical entailment relation

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\(\varphi, \psi\) agent terms, with one of them a subterm of the other.

\[
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\]

\[
\varphi \models_{K} \psi \quad \text{if} \quad \varphi \models_{\langle K, H \rangle} \psi \quad \text{for all} \quad H
\]
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\end{align*}
\]

Believing in \( \varphi \) forces the observer to not disbelieve in \( \psi \)
Examples

\[ A_f \xrightarrow{-} A_c \]

\[ S \]

\[ K(p) \]

\[ A_m \rightarrow A_f \xrightarrow{-} A_c \]

\[ S \]

\[ K(\neg p) \]
Examples

\[
K(p)
\]

\[
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\[
\begin{array}{|c|c|c|}
\hline
H(S) & St_H(K(\neg p)) & St_H(K(p)) \\
\hline
(0, 1] & -1 & 1 \\
0 & 0 & 0 \\
[-1, 0) & 1 & -1 \\
\hline
\end{array}
\]

\[ \neg p \not\models_K p \]
Examples
Examples

\( K(p) \)

\( K(q) \)

\( K(p \land q) = K(p) \circ K(q) \)
A logical language

Examples

\[ K(p) \]

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We have

\[ p \land q \models_K p \]

Similarly:

\[ p \models_K p \lor q \]

\[ \neg p \land \neg q \models_K \neg(p \lor q) \]

\[ \neg(p \lor q) \models_K \neg(p \land \neg q) \]
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Summary
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- DLGs for representing communication situations.
Summary

- DLGs for representing communication situations.
- Assertion network semantics.
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- DLGs for representing communication situations.
- Assertion network semantics.
- A logical language modelling AN.
Summary

- DLGs for representing communication situations.
- Assertion network semantics.
- A logical language modelling AN.
- Simple graph operations defined and analyzed.
Further work
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- Alternative definitions of $\psi$. 
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- Generalization of the whole scenario (too much restrictions has been imposed).
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- Another graph operations (relating facts instead of agents)
Further work

- Alternative definitions of $\psi$.
- Generalization of the whole scenario (too much restrictions has been imposed).
- Another graph operations (relating facts instead of agents).
- A more universal entailment relation rather than the very contextual one given here.
Thanks