Scalar Implicatures and Implicatures of Irrelevant Answers

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Core Examples

Example 1 (Out of Petrol)

I is standing by an obviously immobilized car and is approached by E, after which the following exchange takes place:

I: I am out of patrol.
E: There is a garage round the corner. \((G)\)
\(\rightarrow\) The garage is open. \((H)\)
Core Examples

Example 2 (Bus Ticket)
An email was sent to all employees that bus tickets for a joint excursion have been bought and are ready to be picked up. By mistake, no contact person was named. Hence, I asks one of the secretaries:

I: Where can I get the bus tickets for the excursion?
E: Ms. Müller is sitting in office 2.07. (M)
→ Bus tickets are available from Ms. Müller. (H)
Outline

The Optimal Answer (OA) Model

Implicatures in the OA Model

Optimal Completion
Section I

The Optimal Answer (OA) Model
General Situation

Expert $E$ answers

\[ \uparrow \text{expectations of } E \quad (\Omega, P_E) \]

\[ \downarrow \]

\[ \bullet \]

\[ \bullet \quad A \rightarrow \]

$I$ decides for action

\[ \uparrow \text{expectations of } I \quad (\Omega, P_I) \]

\[ \downarrow \]

\[ \bullet \quad a \rightarrow \]

Evaluation

\[ \uparrow \text{utility function} \quad u(a, v) \]
We consider situations in which:

- A person \( I \), called inquirer, has to solve a decision problem \( \langle (\Omega, P), A, u \rangle \).
- A person \( E \), called expert, provides \( I \) with information that helps solving \( E \)’s decision problem.
- \( P_E \) represents \( E \)’s expectations about \( \Omega \) at the time when \( E \) answers.
Decision Problems

Definition 3
A decision problem is a triple \( \langle (\Omega, P), \mathcal{A}, u \rangle \) such that:

- \( (\Omega, P) \) is a finite probability space,
- \( \mathcal{A} \) a finite, non-empty set, and
- \( u : \mathcal{A} \times \Omega \rightarrow IR \) a function.

\( \mathcal{A} \) is called the action set, and its elements actions. \( u \) is called a payoff or utility function.
Support Problems

Definition 4 (Support Problem)

\( \sigma = \langle \Omega, P_E, P_I, A, u \rangle \) is a **support problem** if

- \( (\Omega, P_E) \) is a finite probability space, and
- \( \langle (\Omega, P_I), A, u \rangle \) a decision problem.

We assume:

\[ \forall X \subseteq \Omega \ P_E(X) = P_I(X|K) \text{ for } K = \{ v \in \Omega \mid P_E(v) > 0 \}. \] (1)
The Inquirer’s Decision Situation

Optimising expected utilities of actions

The expected utility of an action \( a \) is defined by:

\[
EU(a) = \sum_{v \in \Omega} P(v) \times u(a, v).
\]  

(2)

After learning \( A \), the inquirer optimises the conditional expected utility:

\[
EU_I(a|A) = \sum_{v \in \Omega} P_I(v|A) \times u(a, v).
\]  

(3)

Hence, he will choose his actions from the set:

\[
B(A) := \{ a \in \mathcal{A} \mid \forall b \in \mathcal{A} \; EU_I(b|A) \leq EU_I(a|A) \}.
\]  

(4)
The Expert’s Decision Situation

Optimising expected utilities of answers

If there exists for each answer \( A \) a unique optimal choice \( a_A \in B(A) \), then the expected utility of an answer is defined as:

\[
EU_E(A) := \sum_{v \in \Omega} P_E(v) \times u(v, a_A) = EU_E(a_A). \tag{5}
\]

If the inquirer’s choice is not unique, then let \( h(\cdot | A) \) be a probability distribution over \( B(A) \) representing the inquirer’s choice:

\[
h(a|A) > 0 \Rightarrow a \in B(A). \tag{6}
\]

The expert has to optimise:

\[
EU_E(A) := \sum_{a \in B(A)} h(a|A) \times EU_E(a). \tag{7}
\]
Maxim of Quality: Be truthful!

This restricts the expert’s answers to:

\[ Adm_\sigma := \{ A \subseteq \Omega \mid P_E(A) = 1 \} \]  

(8)

Hence, the set of optimal answers is provided by:

\[ Op_\sigma := \{ A \in Adm_\sigma \mid \forall B \in Adm_\sigma \ EU_E(B) \leq EU_E(A) \}. \]  

(9)
Section II

Implicatures in the OA Model
Implicatures

What is an implicature?
“... what is implicated is what is required that one assume a speaker to think in order to preserve the assumption that he is observing the Cooperative Principle (and perhaps some conversational maxims as well), ...”
Definition of Implicatures

We write $A \rightarrow H$ if an utterance of $A$ implicates proposition $H$.

Definition 5 (Implicature)

Let $\sigma = \langle \Omega, P_E, P_I, A, u \rangle$ be a given support problem, $\sigma \in \hat{\mathcal{S}} \subseteq \mathcal{S}$. For $A, H \in \mathcal{P}(\Omega)$, $A \in \text{Op}_\sigma$ we define:

$$A \rightarrow H : \iff \forall \hat{\sigma} \in [\sigma]_{\hat{\mathcal{S}}}: \ A \in \text{Op}_{\hat{\sigma}} \rightarrow P_{\hat{E}}(H) = 1,$$

(10)

with $[\sigma]_{\hat{\mathcal{S}}}$ the set of all support problems that only differ in $P_E$ from $\sigma$. 
Special Case
Expert knows optimal Action

Let $O(a)$ be the set of all worlds where $a$ is an optimal action:

$$ O(a) := \{ w \in \Omega | \forall b \in A u(a, w) \geq u(b, w) \}. \quad (11) $$

As a special case, we find:

**Lemma 6**

Let $\hat{S}$ be the set of all support problems with

$\exists a \in A P_E(O(a)) = 1$. Let $\sigma \in \hat{S}, A, H \subseteq \Omega, A \in Op_\sigma$, and $A^* := \{ w \in A | P_I(w) > 0 \}$. Then:

$$ A \Rightarrow H \text{ iff } \forall a \in B(A) : A^* \cap O(a) \subseteq H. \quad (12) $$
Special Case

Application

Example 7

A: I am out of petrol.
B: There is a garage round the corner. (G) +> The garage is open. (H)

<table>
<thead>
<tr>
<th>$\Omega$</th>
<th>$G$</th>
<th>$H$</th>
<th>go-to-g</th>
<th>search</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_1$</td>
<td>+</td>
<td>+</td>
<td>1</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>$w_2$</td>
<td>+</td>
<td>-</td>
<td>0</td>
<td>$\varepsilon$</td>
</tr>
<tr>
<td>$w_3$</td>
<td>-</td>
<td>-</td>
<td>0</td>
<td>$\varepsilon$</td>
</tr>
</tbody>
</table>

- Assumption: $EU_{i}(\text{go-to-g}|G) > \varepsilon$.
- $O(\text{go-to-g}) = \{w_1\} = H$.
- Hence, $G +> H$. 
Section III

Optimal Completion
Special Case
All expert types possible

Let \( \hat{S} \) be such that:

1. \( \forall \theta \subseteq \Omega \ \exists \sigma \in \hat{S} : \theta = \{ v \in \Omega \mid P_E^\sigma(v) > 0 \} \),
2. \( A \subsetneq B \Rightarrow EU_E(A) > EU_E(B) \).

Then:

For all \( H \subseteq \Omega : A \rightarrow H \text{ iff } H = A^* \).
Consequences

- Implicatures arise if not all speaker’s information states are possible.
- Implicatures are only defined for

\[ \{ F | \exists \sigma F \in Op_\sigma \} \]

Question
Are there natural principles that allow to generate implicatures for \( F \)s not in \( \{ F | \exists \sigma F \in Op_\sigma \} \)?
An Example

Schalar Implicatures

1. All of the boys came to the party. \((F_{\forall})\)
2. Some of the boys came to the party. \((F_{\exists})\)
3. Some but not all of the boys came to the party. \((F_{\exists \neg \forall})\)
4. Not all of the boys came to the party. \((F_{\neg \forall})\)
5. None of the boys came to the party. \((F_{\neg \exists})\)

\[ F_{\exists}, F_{\neg \forall} \implies F_{\exists \neg \forall} \]
Observation

1. \( \{ F \mid \exists \sigma F \in \text{Op}_\sigma \} = \{ F_\forall, F_{\neg \exists}, F_{\exists \neg \forall} \} \).

2. \( F_{\exists}, F_{\neg \forall} \not\in \{ F \mid \exists \sigma F \in \text{Op}_\sigma \} \).

3. Some and Not All are sub-forms of Some but not All:

   \[ F_{\exists}, F_{\neg \forall} \prec F_{\exists \neg \forall} \]

Hypothesis

Unused non-optimal sub-forms inherit the interpretation of their optimal super-forms.

Principle of Optimal Completion
Example 8

I: Where can I get the bus tickets for the excursion?

1. $E$: Ms. Müller is sitting in office 2.07. ($F_M$)
2. $E$: Bus tickets are available from Ms. Müller. ($F_H$)
3. $E$: Bus tickets are available from Ms. Müller. She is sitting in office 2.07. ($F_{MH}$)

- $F_M \triangleleft F_{MH}$,
- $F_M \notin \{F \mid \exists \sigma F \in \text{Op}_\sigma\}$,
- $F_{MH} \in \{F \mid \exists \sigma F \in \text{Op}_\sigma\}$. 
What can justify Optimal Completion?

Robustness
Language interpretation should be robust against mistakes by the speaker.

Definition 9 (Trembling Hand)
A trembling hand prefect equilibrium of a finite strategic game is a mixed strategy profile $\sigma$ such that there exists a sequence $(\sigma^k)_{k=0}^\infty$ of completely mixed strategy profiles which converge to $\sigma$ such that $\sigma_i$ is a best response to each $\sigma^k_i$ [Osborne & Rubinstein(1994), Def. 248.1].
Which Trembles Count?

Not all mistakes equally probable:

1. If speaker is expert, he knows whether A(all), A(some but not all), or A(none);
2. Hence, A(some, if not all) is sub-optimal and entails A(all);
   \[\Rightarrow\] Assumption: A(some, if not all) will not occur as mistake for A(all).

Only sub-forms of optimal forms count:

1. A(some) is sub-form of A(some but not all) and A(some, if not all);
2. If the speaker is expert, then A(some, if not all) is sub-optimal;
   \[\Rightarrow\] A(some) can only be optimally completed to A(some but not all).
Definition 10
We say that, for a support problem $\sigma$, a form $E$ can be **optimally completed** to form $F$, $oc(\sigma, E, F)$, iff

1. $F \in Op_\sigma$,
2. $E \not\in Op_\sigma$,
3. $P_\sigma^E(\lbrack E \rbrack) = P_\sigma^E(\lbrack F \rbrack) = 1$, 
4. $E \rhd F$. 
Putting Noise into Speakers’ strategies

An epsilon downward approximation of a mixed speaker’s strategy $s$ is a probability distribution $s^\epsilon$ on $\sigma \times F$ such that:

1. $s^\epsilon(\cdot|\sigma)$ is defined for
   - optimal forms in $\sigma$,
   - sub-forms $E$ of optimal forms $F$ such that:
     - $E$ is true in $\sigma$,
     - $F$ is the only optimal super-form of $E$.

2. for optimal forms $F$,

   $$s^\epsilon(F|\sigma) = (1 - \epsilon)s(F|\sigma)$$

3. for all sub-optimal forms $E$ of $F$,

   $$s^\epsilon(E|\sigma) = \epsilon n^{-1} s(F|\sigma),$$

with $n$ the number of forms that can only be optimally completed to $F$. 
Definition 11 (Downward Perfect Equilibrium)

A strategy pair \((s, h)\) is a downward (trembling hand) perfect Bayesian equilibrium iff \((s, h)\) is a perfect Bayesian equilibrium and if for every sequence \((s^\epsilon_n)_{n>0}\) of epsilon downward approximations of \(s\) with \(\epsilon_n \to 0\), there exists an \(N\) such that for all \(n \geq N\) \(h\) is a perfect Bayesian best response to \(s^\epsilon_n\).
The Principle of Optimal Completion

The hearer can interpret a non-optimal utterance $E$ in $\sigma$ iff

1. $\exists \sigma \in [\sigma] \exists F \in \mathcal{F} \ \text{oc}(\sigma, E, F)$,

2. $\forall \sigma, \sigma' \in [\sigma] \forall F, F' \in \mathcal{F}$:

$$\text{oc}(\sigma, E, F) \land \text{oc}(\sigma', E, F') \rightarrow F = F'.$$

Then, utterance $E$ implicates $A$, $E \Rightarrow A$, iff

$$\forall \sigma \in [\sigma] \forall F \in \mathcal{F} \ (\text{oc}(\sigma, E, F) \rightarrow P_\sigma^E(A) = 1),$$

which is equivalent to

$$\forall \sigma \in [\sigma] \forall F \in \mathcal{F} \ (\text{oc}(\sigma, E, F) \rightarrow F \Rightarrow A),$$


