Alleged Assassins

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TbiLLC 2011 – Kutaisi, Georgia
Outline

1. The Problem of Modal Modification
2. Solutions in terms of Procedural Semantics
3. Transparent Intensional Logic
4. Modal Constructive Type Theory
5. Conclusions
1. The Problem of Modal Modification

2. Solutions in terms of Procedural Semantics

3. Transparent Intensional Logic

4. Modal Constructive Type Theory

5. Conclusions
a is an **alleged** assassin

?
a is an alleged assassin

what is the logical structure of the premise?
what follows as conclusion?
Let $M$ be a modifier and $F$ a property. Then $(MF)$ is the result of the procedure of applying the function $M$ to the argument $F$. 
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A full semantic theory of modification must include the following variants:

- **Subsective:** $(M'F)a \rightarrow Fa$
- **Privative:** $(M''F)a \rightarrow \neg Fa$
- **Intersective:** $(M'''F)a \rightarrow M^*a \land Fa$
- **Modal:** $M''''$ oscillates between subsection and privation
3 Negative Characterizations of $M'''$

\[
\frac{(MF)_c x}{M^*_c x \land F_c x}
\]
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\frac{(MF)_c x}{M^*_c x \land F_c x}
\]

\[
F_c x \leftrightarrow G_c x \quad F_c x \rightarrow (MF)_c x
\]

\[
G_c x \rightarrow (MG)_c x
\]
3 Negative Characterizations of $M'''$

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\frac{(MF)_c x}{M^*_c x \land F_c x}
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\[
F_c x \leftrightarrow G_c x \quad F_c x \rightarrow (MF)_c x
\]

\[
G_c x \rightarrow (MG)_c x
\]

Fails to validate either of $Fa$, $\neg Fa$ as conclusion.
Task

A positive characterization of modal modification.
A solution to privative modification

[Primiero and Jespersen, 2010] offers two analyses of privative modification using two variants of procedural semantics:

Realism: Tichý’s Transparent Intensional Logic
Idealism: Martin-Löf’s Constructive Type Theory
A solution to privative modification

[Primiero and Jespersen, 2010] offers two analyses of *privative modification* using two variants of *procedural semantics*:

- **Realism**: Tichý’s Transparent Intensional Logic
- **Idealism**: Martin-Löf’s Constructive Type Theory

Common basic idea is to analyze *modal modification* in terms of *possibility/contingency*:

- TIL: alethic
- CTT: epistemic
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The Common Core

1. a notion of *construction*
2. a *functional* language
3. a *typed* universe
4. an *interpreted* syntax
What Distinguishes TIL from CTT

<table>
<thead>
<tr>
<th></th>
<th>TIL</th>
<th>CTT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Semantics</td>
<td>model-theoretic</td>
<td>proof-theoretic</td>
</tr>
<tr>
<td>Modifier</td>
<td>property to property</td>
<td>set to set</td>
</tr>
</tbody>
</table>

Jespersen, Primiero (Ostrava – Ghent)
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Basic and Functional Types

- **Ground Types**: $o$, $ι$, $τ$, $ω$

- **Property**: $(o_ι)_τω$

- **Property modifier**: $((o_ι)_τω (o_ι)_τω)$

- **Proposition**: $o_τω$

- **Propositional modifier**: $o_τ_ω o_τ_ω$
“a is an alleged assassin”

\[ \lambda w \lambda t \left[ [Alleged \, Assassin]_{wt} \, a \right] \]
The speech act of allegation

\[
\lambda w \lambda t \left[ \text{Alleges}_{wt} b \ \lambda w' \lambda t' [F_{w't'} a] \right] \\
\lambda w \lambda t \left[ \exists x [\exists P [\text{Alleges}_{wt} x P]] \right] \\
\text{EG}
\]

“b alleges that a is an F”
“somebody alleges something”
Introduction rule for *Alleged*

\[ \lambda f[[\text{Alleged } f]_{wt} a] = \lambda f[\exists x[\text{Alleges}_{wt} x \lambda w \lambda t[f_{wt} a]]] \]

“being a property that \( a \) is alleged to have equals being a property that somebody alleges \( a \) to have”
Elimination Rule for \textit{Alleged}

\[
[[\text{Alleged Assassin}]_{wt} \ a] \\
\exists w' [\exists t' [\text{Assassin}_{w't'} \ a]] \land \exists w'' [\exists t'' \neg [\text{Assassin}_{w''t''} \ a]]
\]
Introduction rule for *Allegedly*

\[ \lambda P[Allegedly\ P] = \lambda P[\lambda w \lambda t[\exists x[Alleges_{wt} x P]]] \]

“being an alleged proposition equals being a proposition that somebody alleges”
Elimination rule for *Allegedly*

\[
\frac{[\text{Allegedly } P]_{wt}}{\exists w'[\exists t[P_{w't'}]] \land \exists w''[\exists t''[\neg P_{w''t''}]]}
\]
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Two initial comments

1. Given the judgemental structure of formulas in CTT, we can model only the propositional modifier:
   - from ‘a is an alleged assassin’ to ‘Allegedly, a is an assassin’

2. The standard constructive syntax does not allow to deal with the contingency required by modal modifiers:
   - an extended language is required
Definition (Alphabet)

The syntax is defined by the following alphabet:

\[ \mathcal{K} : \{ \text{type}, \text{type}_{inf} \} \] (verifiers, possibly terminating processes)

**Types**: \( A \mid ⊥ \mid A \land B \mid A \lor B \mid A \rightarrow B \mid A \supset B \).

**Terms**: \( x_i \mid a_i \mid (a_i, b_j) \mid (x_i(b_j)) \mid a_i(b_j) \).

**Contexts**: \( Γ_i \mid Δ_i \mid □_iΓ \mid ◊_iΓ \)

**Judgements**: \( Δ_i;Γ_i \vdash A \text{ type} \mid □_i(A \text{ true}) \mid ◊_i(A \text{ true}) \mid o_{i,j}Γ \vdash o_{i,j}(A \text{ true}) \).
Definition (Alphabet)

The syntax is defined by the following alphabet:

\[ \mathcal{K} : \{ \text{type}, \text{type}_{inf} \} \] (verifiers, possibly terminating processes)

Types := \( A \mid \bot \mid A \land B \mid A \lor B \mid A \rightarrow B \mid A \supset B \).

Terms := \( x_i \mid a_i \mid (a_i, b_j) \mid (x_i(b_j)) \mid a_i(b_j) \).

Contexts := \( \Gamma_i \mid \Delta_i \mid \square_i \Gamma \mid \Diamond_i \Gamma \).

Judgements := \( \Delta_i; \Gamma_i \vdash A \text{ type} \mid \square_i (A \text{ true}) \mid \Diamond_i (A \text{ true}) \mid \circ_{i,j} \Gamma \vdash \circ_{i,j} (A \text{ true}) \).
Modal Modification Rule: Introduction

Allegedly \([a \text{ is an assassin}]\)

\[
\text{Assassin type}[\Gamma] \quad \text{Property}_i \text{ type}_{\text{inf}} \in \Gamma \quad \text{Alleged}(x)[x : \text{Assassin}]
\]

\[\Box \Gamma, \Diamond (\text{Property}_i) \vdash a : \text{Assassin}[x_i/p_i : \text{Property}_i]\]
It is proven that [a is an assassin]

\[ \Box \Gamma, \Diamond (Property_i) \vdash a : \text{Assassin}[x_i/p_i : Property_i] \quad p_i : Property_i \]

\[ \Box (\Gamma, p_i : Property_i) \vdash a : \text{Assassin} \]

\[ A \text{ type}_{inf} \quad x : A \vdash B \text{ type}_{inf} \quad a : A \]

\[ (x(b))(a) = b[a/x] : B \text{ type}[a/x] \] \(\beta\)-conversion
The allegation that \([a \text{ is an assassin}]\) is false.

\[
\Box \Gamma, x_i : Property_i \vdash a : \text{Assassin}[x_i/p_i:Property_i] \quad p_i : Property_i \rightarrow \bot
\]

\[
a : \text{Assassin} \rightarrow \bot
\]
The Problem of Modal Modification

Solutions in terms of Procedural Semantics

Transparent Intensional Logic

Modal Constructive Type Theory

Conclusions
Summary of this presentation

1. Oscillation between subsection and privation

2. Alethic vs. Epistemic Possibility

3. For TIL, if \( (M''''F)a \) is true, then at some pair \( wt \) (empirical parameters) \( Fa \) is true and at another such pair \( Fa \) is false

4. For CTT, if \( (M''''F)a \) true is an admissible judgment to make, then conditions for \( Fa \) true are known to be satisfiable, but not all are asserted as verified

