A complete axiomatization of Euclidean strong non-contingency logic

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Abstract. This article provides a complete axiomatization for strong non-contingency logic over Euclidean frames, whose completeness proof is nontrivial. Our result answers an open question raised in [3].

1 Introduction

Past decades have witnessed a variety of research on logics with a sole primitive modality that is essentially a combination of another modality and boolean connectives. For instance, in non-contingency logic [10,1,7,8,2,4,5], a formula is noncontingent iff it is either necessarily true or necessarily false, whereas a formula is contingent iff it is possibly true and also possibly false, in symbol, $\Delta\varphi = \Box\varphi \vee \Box\neg\varphi$, $\nabla\varphi = \Diamond\varphi \wedge \Diamond\neg\varphi$; in the logic of essence and accident [9,11], a formula is essential iff once it is true, it is necessarily true, while a formula is accident iff it is true but possibly false, in symbol, $\circ\varphi = \varphi \to \Box\varphi$, $\bullet\varphi = \varphi \wedge \Diamond\neg\varphi$; in the logic for false belief [12], φ is a false belief iff φ is false but believed, in symbol, $W\varphi = \neg\varphi \wedge B\varphi$. Despite being definable with known modalities such as necessity/belief, these modalities have philosophical interests in their own right, and deserve to be studied independently.

Recently, Fan [3] has introduced the notion of *strong non-contingency* by saying that a formula is strongly non-contingent iff it is necessarily true when it is true and it is necessarily false when it is false. This notion is related to Hintikka's treatment of question embedding verbs like 'know', 'remember' in [6]. According to his treatment, "Mary knows (remembers) whether it is raining" is equivalent to "if it is raining, then Mary knows (remembers) it is raining, and if it is not raining, then Mary knows (remembers) it is not raining". Just as necessity means (propositional) knowledge in the setting of epistemic logic, strong non-contingency means knowledge whether in the sense of Hintikka's aforementioned treatment.

As shown in [3], the logic with strong non-contingency as a sole primitive modality, a non-normal modal logic, is less expressive than standard modal logic on various classes of models, and cannot define many usual frame properties including Euclideanity. This may invite technical difficulties and novelties in completely axiomatizing this new logic over various frames. [3] has completely axiomatized different modal logics of strong non-contingency, and leave open the question of the complete axiomatization of the modal logic of strong non-contingency determined by the class of all Euclidean frames. In this note, we answer the open question.

2 Formal definitions

We adopt the notation from [3].

Syntax Let ATO be a countable set of atoms (p, q, etc). The set FOR of all formulas (φ, ψ, etc) is inductively defined as follows:

- $\varphi ::= p \mid \bot \mid \neg\varphi \mid (\varphi \vee \psi) \mid \blacktriangle\varphi$.

The formula $\blacktriangledown\varphi$ is obtained as an abbreviation for $\blacktriangledown\varphi ::= \neg\blacktriangle\varphi$. The formulas φ^0, φ^1 and φ^2 are respectively obtained as abbreviations for $\blacktriangledown\varphi \wedge \blacktriangle\blacktriangle\varphi$, $\varphi \wedge \blacktriangle\varphi$ and $\neg\varphi \wedge \blacktriangle\varphi$.

Semantics A frame is a structure of the form $\mathcal{F} = (W, R)$ where W is a nonempty set of states (s, t, etc) and R is a binary relation on W. A model based on a frame $\mathcal{F} = (W, R)$ is a structure of the form $\mathcal{M} = (W, R, V)$ where V is a valuation on W, i.e. a function associating to each atom p a set $V(p)$ of states. Atoms and Boolean connectives being classically interpreted, we inductively define the truth of $\varphi \in FOR$ in model $\mathcal{M} = (W, R, V)$ at state $s \in W$ ($\mathcal{M}, s \models \varphi$) as follows:

- $\mathcal{M}, s \models \blacktriangle\varphi$ iff if $\mathcal{M}, s \models \varphi$ then $\mathcal{M}, t \models \varphi$ for all states $t \in W$ such that sRt and if $\mathcal{M}, s \not\models \varphi$ then $\mathcal{M}, t \not\models \varphi$ for all states $t \in W$ such that sRt.

Let $\mathcal{F} = (W, R)$ be a frame. We shall say $\varphi \in FOR$ is $\mathcal{F}$-valid ($\mathcal{F} \models \varphi$) iff for all models $\mathcal{M}$ based on $\mathcal{F}$ and for all states $s \in W$, $\mathcal{M}, s \models \varphi$. Let $\mathcal{C}$ be a class of frames. We shall say $\varphi \in FOR$ is $\mathcal{C}$-valid ($\mathcal{C} \models \varphi$) iff for all frames $\mathcal{F}$ in $\mathcal{C}$, $\mathcal{F} \models \varphi$. The logic determined by a class $\mathcal{C}$ of frames ($Log(\mathcal{C})$) is the set of $\varphi \in FOR$ such that $\mathcal{C} \models \varphi$.

Generated subframes The subframe of a frame $\mathcal{F} = (W, R)$ generated by a state $s \in W$ is the restriction of $\mathcal{F}$ to the states in $R^\star(s)$ where $R^\star$ is the reflexive transitive closure of R; in other words, $R^\star = \bigcup_{n \in \mathbb{N}} R^n$. We shall say that a frame $\mathcal{F} = (W, R)$ is generated iff there exists a state $s \in W$ such that $R^\star(s) = W$.

Proposition 1 (Generated Subframe Lemma). *Let $\mathcal{F} = (W, R)$ and $\mathcal{F}' = (W', R')$ be frames. If $\mathcal{F}'$ is a subframe of $\mathcal{F}$ generated by a state $s \in W$ then for all $\varphi \in FOR$, if $\mathcal{F} \models \varphi$ then $\mathcal{F}' \models \varphi$.*

Bounded morphisms Let $\mathcal{F} = (W, R)$ and $\mathcal{F}' = (W', R')$ be frames. A function μ associating to each state $s \in W$ a state $\mu(s) \in W'$ is said to be a bounded morphism from $\mathcal{F}$ to $\mathcal{F}'$ iff the following conditions hold:

- If sRt and $s \neq t$ then $\mu(s)R'\mu(t)$.
- If $\mu(s)R't'$ and $\mu(s) \neq t'$ then there exists a state $t \in W$ such that sRt and $\mu(t) = t'$.

We shall say that $\mathcal{F}'$ is a bounded morphic image of $\mathcal{F}$ iff there exists a surjective bounded morphism from from $\mathcal{F}$ to $\mathcal{F}'$.

Proposition 2 (Bounded Morphism Lemma). *Let $\mathcal{F} = (W, R)$ and $\mathcal{F}' = (W', R')$ be frames. If $\mathcal{F}'$ is a bounded morphic image of $\mathcal{F}$ then for all $\varphi \in FOR$, if $\mathcal{F} \models \varphi$ then $\mathcal{F}' \models \varphi$.*

Euclidean frames A frame $\mathcal{F} = (W, R)$ is said to be Euclidean iff for all states $s, t, u \in W$, if sRt and sRu then tRu and uRt. Let $\mathcal{C}_{euc}$ be the class of all Euclidean frames.

Proposition 3. *Let $\mathcal{F} = (W, R)$ be an Euclidean frame. For all states $s \in W$, $R^\star(s) = \{s\} \cup R(s) \cup R(R(s))$.*

Proof. By the definition of $R^\star$, we need only show

$$(*) \quad \text{for all } n \geq 3, R^n(s) \subseteq R(R(s)).$$

We prove this by induction on $n \geq 3$.

- $n = 3$. Suppose $t \in R^3(s)$, then there exist u_1, u_2 such that sRu_1Ru_2Rt. Since R is Euclidean, u_1Ru_1, and then u_2Ru_1. From this and u_2Rt, by using Euclideanity of R, we obtain u_1Rt, then $t \in R(R(s))$.
- Inductively hypothesize (IH) that $(*)$ holds for $n = k$, we show it also holds for $n = k+1$. Assume $t \in R^{k+1}(s)$, then there is a u such that sR^kuRt. By $u \in R^k(s)$ and IH, we derive $u \in R(R(s))$, and thus $t \in R^3(s)$. By a similar argument as the case $n = 3$, we conclude that $t \in R(R(s))$.

Let A, B and C be pairwise disjoint sets. Let $\mathcal{F}^A_{B,C} = (W^A_{B,C}, R^A_{B,C})$ be the frame such that $W^A_{B,C} = A \cup B \cup C$ and $R^A_{B,C} = \{(s,t)$: either $s \notin A$ and $t \in C$, or $t \in B\}$.

Proposition 4. *Let A, B and C be pairwise disjoint sets. Then $\mathcal{F}^A_{B,C}$ is in $\mathcal{C}_{euc}$.*

Proof. Suppose $\mathcal{F}^A_{B,C}$ is not in $\mathcal{C}_{euc}$. Let $s, t, u \in W^A_{B,C}$ be states such that $sR^A_{B,C}t$, $sR^A_{B,C}u$ and either not $tR^A_{B,C}u$, or not $uR^A_{B,C}t$. Without loss of generality, suppose not $tR^A_{B,C}u$. Since $sR^A_{B,C}t$, therefore either $s \notin A$ and $t \in C$, or $t \in B$. Hence, either $t \in C$, or $t \in B$. Since $sR^A_{B,C}u$, therefore either $s \notin A$ and $u \in C$, or $u \in B$. Thus, either $u \in C$, or $u \in B$. Since not $tR^A_{B,C}u$, therefore either $t \in A$, or $u \notin C$. Moreover, $u \notin B$. Since either $t \in C$, or $t \in B$, therefore $u \notin C$: a contradiction.

Proposition 5. *Let $\mathcal{F} = (W, R)$ be a generated Euclidean frame. There exists pairwise disjoint sets A, B and C such that $\mathcal{F}$ is a bounded morphic image of $\mathcal{F}^A_{B,C}$.*

Proof. Let $s \in W$ be a state such that $R^\star(s) = W$. Since $\mathcal{F}$ is Euclidean, therefore $W = \{s\} \cup R(s) \cup R(R(s))$. Let A, B and C be the pairwise disjoint sets defined as follows:

- $A = \{(s, 0)\}$.
- $B = \{(t, 1) : \ t \in R(s)\}$.
- $C = \{(u, 2) : \ u \in R(R(s))\}$.

Let μ be the function associating to each state $x \in W^A_{B,C}$ a state $\mu(x) \in W$ defined as follows:

- $\mu(s, 0) = s$.
- For all $t \in R(s)$, $\mu(t, 1) = t$.
- For all $u \in R(R(s))$, $\mu(u, 2) = u$.

Obviously, μ is a surjective bounded morphism from $\mathcal{F}^A_{B,C}$ to $\mathcal{F}$.

3 Axiomatization

Let $L^{\blacktriangle}_{euc}$ be the least set of formulas closed under the inference rules of modus ponens and uniform substitution, closed under the inference rule $\dfrac{\varphi \rightarrow \psi}{\varphi \wedge \blacktriangle\varphi \rightarrow \psi \wedge \blacktriangle\psi}$, containing all propositional tautologies and containing the following formulas:

$(K1)$ $\blacktriangle\top$,
$(K2)$ $\blacktriangle\neg p \leftrightarrow \blacktriangle p$,
$(K3)$ $\blacktriangle p \wedge \blacktriangle q \rightarrow \blacktriangle(p \wedge q)$,
$(A1)$ $\blacktriangledown(p \wedge q^1) \wedge \blacktriangledown(p \wedge \neg q^1) \rightarrow p$,
$(A2)$ $\blacktriangle(p \wedge q^0) \wedge \blacktriangle(p \wedge q^1) \wedge \blacktriangle(p \wedge q^2) \rightarrow p \vee \blacktriangle p$.

The inference rule $\dfrac{\varphi \rightarrow \psi}{\varphi \wedge \blacktriangle\varphi \rightarrow \psi \wedge \blacktriangle\psi}$ and the formulas $(K1)$, $(K2)$ and $(K3)$ have already been considered in [3].

Proposition 6 (Soundness). *For all $\varphi \in FOR$, if $\varphi \in L^{\blacktriangle}_{euc}$ then $\varphi \in Log(\mathcal{C}_{euc})$.*

Proof. Suppose $(A1)$ is not in $Log(\mathcal{C}_{euc})$. Let $\mathcal{F} = (W, R)$ be an Euclidean frame, $\mathcal{M} = (W, R, V)$ be a model based on $\mathcal{F}$ and $s \in W$ be a state such that $\mathcal{M}, s \not\models \blacktriangledown(p \wedge q^1) \wedge \blacktriangledown(p \wedge \neg q^1) \rightarrow p$. Hence, $\mathcal{M}, s \models \blacktriangledown(p \wedge q^1)$, $\mathcal{M}, s \models \blacktriangledown(p \wedge \neg q^1)$ and $\mathcal{M}, s \not\models p$. Let $t, u \in W$ be states such that sRt, sRu, $\mathcal{M}, t \models q^1$ and $\mathcal{M}, u \not\models q^1$. Thus, $\mathcal{M}, t \models q$, $\mathcal{M}, t \models \blacktriangle q$ and either $\mathcal{M}, u \not\models q$, or $\mathcal{M}, u \not\models \blacktriangle q$. Since $\mathcal{F}$ is Euclidean, sRt and sRu, therefore tRu. Since $\mathcal{M}, t \models q$ and $\mathcal{M}, t \models \blacktriangle q$, therefore $\mathcal{M}, u \models q$. Since either $\mathcal{M}, u \not\models q$, or $\mathcal{M}, u \not\models \blacktriangle q$, therefore $\mathcal{M}, u \not\models \blacktriangle q$. Since $\mathcal{M}, u \models q$, therefore let $v \in W$ be a state such that uRv and $\mathcal{M}, v \not\models q$. Since $\mathcal{F}$ is Euclidean, sRt, sRu and uRv, therefore tRv. Since $\mathcal{M}, t \models q$ and $\mathcal{M}, t \models \blacktriangle q$, therefore $\mathcal{M}, v \models q$: a contradiction.
Suppose $(A2)$ is not in $Log(\mathcal{C}_{euc})$. Let $\mathcal{F} = (W, R)$ be an Euclidean frame, $\mathcal{M} = (W, R, V)$ be a model based on $\mathcal{F}$ and $s \in W$ be a state such that $\mathcal{M}, s \not\models \blacktriangle(p \wedge q^0) \wedge \blacktriangle(p \wedge q^1) \wedge \blacktriangle(p \wedge q^2) \rightarrow p \vee \blacktriangle p$. Hence, $\mathcal{M}, s \models \blacktriangle(p \wedge q^0)$, $\mathcal{M}, s \models \blacktriangle(p \wedge q^1)$, $\mathcal{M}, s \models \blacktriangle(p \wedge q^2)$, $\mathcal{M}, s \not\models p$ and $\mathcal{M}, s \not\models \blacktriangle p$. Let $t \in W$ be a state such that sRt and $\mathcal{M}, t \models p$. Since $\mathcal{M}, s \models \blacktriangle(p \wedge q^0)$, $\mathcal{M}, s \models \blacktriangle(p \wedge q^1)$, $\mathcal{M}, s \models \blacktriangle(p \wedge q^2)$ and $\mathcal{M}, s \not\models p$, therefore $\mathcal{M}, t \not\models p \wedge q^0$, $\mathcal{M}, t \not\models p \wedge q^1$ and $\mathcal{M}, t \not\models p \wedge q^2$. Since $\mathcal{M}, t \models p$, therefore $\mathcal{M}, t \not\models q^0$, $\mathcal{M}, t \not\models q^1$ and $\mathcal{M}, t \not\models q^2$. Thus, $\mathcal{M}, t \not\models \blacktriangle q$ and $\mathcal{M}, t \not\models \blacktriangle\blacktriangle q$. Without loss of generality, suppose $\mathcal{M}, t \models q$. Let $u, v \in W$ be a state such that tRu, tRv, $\mathcal{M}, u \not\models q$ and $\mathcal{M}, v \models \blacktriangle q$. Since $\mathcal{F}$ is Euclidean, sRt, tRu and tRv, therefore vRt and vRu. Since $\mathcal{M}, v \models \blacktriangle q$ and $\mathcal{M}, t \models q$, therefore $\mathcal{M}, v \models q$. Since $\mathcal{M}, v \models \blacktriangle q$, $\mathcal{M}, u \not\models q$ and vRu, therefore $\mathcal{M}, v \not\models q$: a contradiction.

We end this section by giving examples of derivable formulas. Let us consider the following formulas:

$(A3)$ $\blacktriangledown(p \wedge q^0) \wedge \blacktriangledown(p \wedge \neg q^0) \rightarrow p$,
$(A4)$ $\blacktriangledown(p \wedge q^2) \wedge \blacktriangledown(p \wedge \neg q^2) \rightarrow p$,
$(B1)$ $\blacktriangledown(p \wedge \neg q^0) \rightarrow p \vee \blacktriangledown(p \wedge q^1) \vee \blacktriangledown(p \wedge q^2)$,
$(B2)$ $\blacktriangledown(p \wedge \neg q^1) \rightarrow p \vee \blacktriangledown(p \wedge q^0) \vee \blacktriangledown(p \wedge q^2)$,

$(B3)$ $\blacktriangledown(p \wedge \neg q^2) \rightarrow p \vee \blacktriangledown(p \wedge q^0) \vee \blacktriangledown(p \wedge q^1)$,
(E) $p \rightarrow \blacktriangle(p \vee q^0 \vee q^1 \vee q^2)$.

Proposition 7. *$(A3)$, $(A4)$, $(B1)$, $(B2)$, $(B3)$ and (E) are in $L^{\blacktriangle}_{euc}$.*

For all $n \geq 0$ and for all $\alpha :\ i \in \{1, \ldots, n\} \mapsto \alpha(i) \in \{0, 1, 2\}$, let us consider the following formula:

(C^{α}_n) $\bigwedge\{p_i \wedge \blacktriangledown(\neg p_i \wedge \neg q_i^{\alpha(i)}) :\ 1 \leq i \leq n\} \rightarrow \bigwedge\{\blacktriangledown(\neg p_i \wedge \bigwedge\{\neg q_j^{\alpha(j)} :\ 1 \leq j \leq n\}) :\ 1 \leq i \leq n\}$.

Proposition 8. *For all $n \geq 0$ and for all $\alpha :\ i \in \{1, \ldots, n\} \mapsto \alpha(i) \in \{0, 1, 2\}$, (C^{α}_n) is in $L^{\blacktriangle}_{euc}$.*

4 Completeness

Our proof of the completeness of $L^{\blacktriangle}_{euc}$ is based on maximal consistent sets of formulas where "consistency" means "$L^{\blacktriangle}_{euc}$-consistency". If Γ is a set of formulas then let $\square\Gamma = \{\varphi :\ \varphi \wedge \blacktriangle\varphi \in \Gamma\}$. Let Γ_0 be a maximal consistent set of formulas. We consider the following two cases: **(i)** for all $\varphi \in FOR$, $\blacktriangle\varphi \in \Gamma_0$, **(ii)** there exists $\varphi \in FOR$ such that $\blacktriangledown\varphi \in \Gamma_0$. In the former case, let $A = \{(\Gamma_0, 0)\}$, $B = \emptyset$ and $C = \emptyset$. Let $\mathcal{M} = (W^A_{B,C}, R^A_{B,C}, V)$ be the model based on $\mathcal{F}^A_{B,C}$ where V is the valuation on $W^A_{B,C}$ such that for all atoms p, $(\Gamma_0, 0) \in V(p)$ iff $p \in \Gamma_0$.

Proposition 9 (Former case: Truth Lemma). *For all $\varphi \in FOR$, $\mathcal{M}, (\Gamma_0, 0) \models \varphi$ iff $\varphi \in \Gamma_0$.*

Proof. By induction on $\varphi \in FOR$. We only treat the case $\blacktriangle\varphi$. As mentioned, in this case $\blacktriangle\varphi \in \Gamma_0$. Moreover, it is easy to show that $R^A_{B,C} = \emptyset$, from which and the semantics of $\blacktriangle$, it follows immediately that $\mathcal{M}, (\Gamma_0, 0) \models \blacktriangle\varphi$.

In the latter case, let $(\varphi_1, \varphi_2, \ldots)$ be an enumeration, possibly with repetitions, of the set of all $\varphi \in FOR$ such that $\varphi \wedge \blacktriangledown\varphi \in \Gamma_0$. For all $i \geq 1$, let $\Delta_i = \square\Gamma_0 \cup \{\neg\varphi_i\}$. Let $(\psi_1, \psi_2, \ldots)$ be an enumeration, possibly with repetitions, of FOR. Let $(a_1, a_2, \ldots) \in \{0, 1, 2\}^{\omega}$ be such that for all $n \geq 0$ and for all $i \geq 1$, $\Delta_i \cup \{\psi_1^{a_1} \wedge \cdots \wedge \psi_n^{a_n}\}$ is consistent. For all $i \geq 1$, let Δ'_i be a maximal consistent set of formulas such that for all $n \geq 0$, $\Delta_i \cup \{\psi_1^{a_1} \wedge \ldots \wedge \psi_n^{a_n}\} \subseteq \Delta'_i$. Let $\Delta' = \Delta'_1 \cup \Delta'_2 \cup \ldots$. Let $(\chi_1, \chi_2, \ldots)$ be an enumeration, possibly with repetitions, of the set of all $\chi \in FOR$ such that $\chi \wedge \blacktriangledown\chi \in \Delta'$. For all $i \geq 1$, let Λ'_i be a maximal consistent set of formulas such that $\square\Delta' \cup \{\neg\chi_i\} \subseteq \Lambda'_i$. Let $A = \{(\Gamma_0, 0)\}$, $B = \{(\Delta'_i, 1) :\ i \geq 1\}$ and $C = \{(\Lambda'_i, 2) :\ i \geq 1\}$. Let $\mathcal{M} = (W^A_{B,C}, R^A_{B,C}, V)$ be the model based on $\mathcal{F}^A_{B,C}$ where V is the valuation on $W^A_{B,C}$ such that for all atoms p, $(\Gamma_0, 0) \in V(p)$ iff $p \in \Gamma_0$ and for all $i \geq 1$, $(\Delta'_i, 1) \in V(p)$ iff $p \in \Delta'_i$ and $(\Lambda'_i, 2) \in V(p)$ iff $p \in \Lambda'_i$.

Proposition 10 (Latter case: Truth Lemma). *For all $\varphi \in FOR$, $\mathcal{M}, (\Gamma_0, 0) \models \varphi$ iff $\varphi \in \Gamma_0$ and for all $i \geq 1$, $\mathcal{M}, (\Delta'_i, 1) \models \varphi$ iff $\varphi \in \Delta'_i$ and $\mathcal{M}, (\Lambda'_i, 2) \models \varphi$ iff $\varphi \in \Lambda'_i$.*

Propositions 9 and 10 immediately yield the following result:

Proposition 11 (Completeness). *For all $\varphi \in FOR$, if $\varphi \in Log(\mathcal{C}_{euc})$ then $\varphi \in L^{\blacktriangle}_{euc}$.*

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