

# *A Brief History of the Logic of Time*

## *Paradoxes of time and dynamical semantics for temporals*

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ABSTRACT: Philosophical paradoxes about time, from ancient to contemporary times, have been catalysts for the development of logics of time. Zeno’s paradoxes posed questions about the infinite divisibility of time intervals and the coherence of infinitesimals (Salmon, 1980). Aristotle’s puzzle in *On Interpretation* about future contingent sentences such as “*tomorrow there will be a sea-battle*” questioned whether such sentences could coherently be ascribed definite truth-values at the present time (Frede, 1970). Diodorus Cronus’s “*Master Argument*” proving a principle of plenitude in which all possibilities are realized in time raised questions about the interaction between modality and temporality (White et al., 1984). In modern times the great divide between A-series and B-series theories of time began with McTaggart’s (McTaggart, 1908) argument for the “unreality of time”. Einstein’s (Einstein, 1905) theory of special relativity proving the *relativity of simultaneity* seemed to undermine the intuitive A-series theories of time but the B-series theories of time continued to have their own paradoxes (e.g., the moving present or “*now*”). Gödel’s (Gödel, 1946/1995, 1949/1990, 1949/1990, 1952/1990) discovery of values for the equations of Einstein’s General Relativity (Einstein, 1915) allowing for time travel seemed further confirmation of McTaggart’s view of the *non-objectivity* of time.

Resolving these philosophical paradoxes has led to an evolving series of logics of time (Burgess et al., 1982, 1982b; Burgess, 1984; Kuhn, 1989; Prior, 1957, 1968; van Benthem, 1982, 1984, 2010; Ludlow, 2018). These models involve distinguishing between *time* and *tense*, between the *ordering* and *ontology* of time, and *instants* and *interval* as well as calling attention to a wealth of linguistics distinctions (e.g., *durative/punctual/telic/non-telic/static/dynamic updating* semantics). Reichenbach’s (Reichenbach, 1956) incidental, but highly influential, remarks about using three reference points to model tense inspired the alternative development of Prior’s (Prior, 1968) modal tense logics. These formalizations were precise enough to lead to definability theorem (e.g., Kamp’s theorem (Kamp, 1968) that every first-order statement with one free variable is definable on continuous linear order using *since* and *until*) but also to undefinability and incompleteness results. Thomason (Thomason, 1972) proved the incompleteness of tense logic with Löb’s Axiom for the past modal operator  $\blacksquare$  “*it has always in the past up until now*”  $\blacksquare(\blacksquare p) \rightarrow p \rightarrow \blacksquare p$  and the McKinsey Axiom  $\square \diamond p \rightarrow \diamond \square p$  using the future modal temporal operators,  $\diamond$  “*at least once in the future it will be the case that*” and,  $\square$  “*always in the future from now.*” The progressive tense is undefinable in the standard temporal base language (van Benthem, 2010).

## Disciplinary Stages: Logic, Space and Time

	Geometries of Space	Logics of Time and Tense
PARADOXES	Zeno's Paradoxes of space, time and motion [c. 450-430 BC]	Aristotle's Sea Battle, Diodorus Cronos Master Arg. [c. 300 BC]
AXIOMATIZATION OF STANDARD THEORIES	Euclid's Elements [c. 300 BC], Felix Klein's [1872] group-theoretic unification of geometries; Hilbert's axiomatization of geometry [1899]	Aristotle's <i>Physics</i> , Reichenbach's model of tenses [1947]. Kamp's definability theorem [1968]. Prior's modal tense logics [1957-1968]
EMERGENCE OF NON-STANDARD THEORIES	Bolyai-Lobachevskian [c. 1830] and Riemannian [1854] non-Euclidean geometries	Einstein's [1905, 1915] relativity of simultaneity, McTaggart [1908] A-series and B-series theories of time
APPLICATIONS OF NON-STANDARD THEORIES	Minkowski's [1915] geometric model of Einstein's Theory of Relativity	Gödel's [1947, 1952] discovery of non-standard topologies and time travel in relativistic universes
PUZZLING OBSERVATIONS & MONSTROSITIES	Peano Curve [1890]. Cantor's Discontinuum [1883]. Weyl-Tile Conundrum [1949]	Tense and Aspect, (Bennett and Partee [1972]), Expletive Negation with <i>Until</i> , <i>Since</i> , <i>Before</i> , etc.
EMERGENCE OF NEW PARADIGM	Fractal Geometries. [1970s]	Dynamical Semantics. [1990s]

Careful empirical observations in linguistics have also been the catalysts for showing the inadequacies of the standard models of tense as well as raising a host of linguistic puzzles (Bennett & Partee, 1972). Why is the durative but not the punctual reading of “*until*” consistent with negative polarity (e.g., “*Gödel did not marry until his father died*” is grammatical but “*Gödel married until his father died*” is not) (Mar, Manyakina, & Caf-fary, 2015). What explains the puzzling synonymy of the so-called “*expletive negation*” constructions such as “*I miss not seeing you*” or the synonymy (in German, but also in Spanish and Hebrew) of such constructions as “*bevor du nicht dein Zimmer aufgeräumt hast, darfst du nicht fernsehen*”? (Cépeda, n.d.) Partee (Partee, 1984) called attention to sentences such as “*Barbara didn't remember that she forgot to turn off the stove until the colloquium started*” which show that some past sentences are more definite than the usual tense logic operators.

Our brief sketch of a history of the logics of time exhibits an evolving dialectic among philosophy, formal logics, and empirical linguistics (Prior, 1957). This pattern of development is shared by the history of other disciplines such as the evolution of the geometries of space (van Benthem, 2010; Mar, 2017) and extensions of classical treatments of the semantic paradoxes into dynamical semantics revealing chaos and fractal images (Mar & Grim, 1991; Mar & St. Denis, 1999; Mar, 2017, 2014, 2006, 2001) .

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