This is a report on work in progress. We consider the incremental construction of H_N, the free Heyting algebra on N generators (or the Lindenbaum algebra for the intuitionistic propositional logic on N variables). This construction which was first described by Ghilardi [Gh92] and was revisited in [B&G11] is an instance of the construction discussed by Sam van Gool in his talk, see also [C&vG]. We observe that each of the surjections in the dual inverse limit system of finite posets has an upper adjoint so that the system is both an inverse limit system and a direct limit (colimit) system of finite posets.

We claim that the direct limit in the category of posets yields the N universal model, Γ_N, whereas the inverse limit of the system taken in the category of ordered topological spaces yields the Esakia space X_N of H_N. Because the two system are related by the fact that the maps are adjoint to each other, Γ_N sits naturally as a subposet of the order-reduct of X_N. Furthermore it follows from the construction that H_N may be embedded in the upset lattice of Γ_N yielding the well-known representation $H_N \hookrightarrow \mathcal{U}(\gamma_N)$ of intuitionistic propositional logic on N variables as the admissible upsets of the N universal model.

In joint work with Serge Grigorieff and Jean-Eric Pin we have identified a topological setting for the study of set representations of distributive lattices (with additional operations), namely these may be viewed as certain (relational) ordered uniform spaces known as Pervin spaces [GGP10]. In this setting, the representation of a distributive lattice L in its Priestley space is obtained as the ordered uniform space completion of any set representation $L \hookrightarrow \mathcal{U}(X)$ of L in an upset lattice. Applying this in the case at hand we see that:

The ordered uniform space completion of the Pervin space corresponding to the set-representation $H_N \hookrightarrow \mathcal{U}(\gamma_N)$ of H_N in the upset lattice of the N universal model is the Esakia space of H_N.

If time permits, we will discus the applicability of this method for obtaining universal-like models for algebraic logics corresponding to varieties of lattices with additional operations in general.

References

[B&G11] Nick Bezhanishvili and Mai Gehrke, Finitely generated free Heyting algebras via Birkhoff duality and coalgebra. To appear in *Logical Methods in Computer Science*, 2011. Available online at http://www.math.ru.nl/ mgehrke.
[C&vG] Dion Coumans and Sam van Gool. Constructing the Lindenbaum algebra for a logic step-by-step using duality. To appear in the *Proceedings of PhDs in Logic III*, 2011. Available online at http://www.math.ru.nl/ vangool/
[GGP10] Mai Gehrke, Serge Grigorieff, and Jean-Eric Pin, A topological approach to recognition, Samson Abramsky et al. (Eds.): *Automata, Languages and Programming, 37th International Colloquium* (ICALP 2010), *Lecture Notes in Computer Science* **6199** (2) (2010),151-162.
[Gh92] Silvio Ghilardi, Free Heyting algebras as bi-Heyting algebras. *Math. Rep. Acad. Sci. Canada* **XVI** (6) (1992), 240-244.