

# Frank Plumpton Ramsey



## The normative/descriptive distinction

Descriptive decision theory is concerned with how agents *actually* act  
↪ cognitive and behavioural psychology, economics

Normative decision theory is concerned with how agents *ought* to act  
↪ philosophy

## Dual role of representation theorems in decision theory

Two different foundational roles:

- 1 *Characterising* preferences, utilities, degrees of belief.
- 2 *Justifying* normative rules e.g. have probabilistic degrees of belief and act as expected utility maximisers.

## Ramsey's aims in 'Truth and Probability'

*'In this essay the Theory of Probability is taken as a branch of logic, the logic of partial belief' (Ramsey 1931, p.156)*

*'The subject of our inquiry is the logic of partial belief, and I do not think we can carry it far unless we have at least an approximate notion of what partial belief is, and how, if at all, it can be measured' (Ramsey 1931, p.157)*

*the degree of belief is like a time interval; it has no precise meaning unless we specify how it is to be measured (Ramsey 1931, 167)*

*"the degree of a belief is a causal property of it, which we can express vaguely as the extent to which we are prepared to act on it" (1931, 169)*

# Building blocks: preferences

Set of worlds  $\Omega = \{\omega_1, \omega_2, \dots\}$

*"different possible totalities of events ... the ultimate organic unities" (1931, 177-8)*

Set of conditional prospects  $\Gamma$

Conditional prospects are statements of the form - if  $P$  then  $\omega_1$ , if  $\neg P$  then  $\omega_2$ , for a given proposition  $P$ . I will denote such conditional prospects as  $(\omega_1, P; \omega_2)$ .

Preferences

Ramsey now needs to elicit a complete preference relation,  $\succeq$ , on  $\Omega \cup \Gamma$ .

## Building blocks: key assumption

*“we act in the way that we think most likely to realize the objects of our desires, so that a person’s actions are completely determined by his desires and opinions... This theory cannot be made adequate to all the facts, but it seems to me a useful approximation to the truth particularly in the case of our self-conscious or professional life, and it is presupposed in a great deal of our thought.” (1931, 173)*

### Formal embodiment of the assumption

There are two real-valued functions, a desirability function *des* and a belief function *bel* such that:

- 1 For a conditional prospect its expected desirability is given by:

$$des((\omega_1, P; \omega_2)) = des(\omega_1)bel(P) + des(\omega_2)bel(\neg P).$$

- 2

$$bel(\neg P) = 1 - bel(P).$$

## Step 1: Elicit a complete $\succeq$ on $\Omega \cup \Gamma$

### End goal

Ramsey first needs to construct  $des$  such that for all  $X, Y \in \Omega \cup \Gamma$ :

$$des(X) \geq des(Y) \Leftrightarrow X \succeq Y.$$

Then, use this  $des$  to determine a unique  $bel$ .

### Ramsey's method

*"If ... we had the power of the Almighty, and could persuade our subject of our power, we could, by offering him options, discover how he placed in order of merit all possible courses of the world. In this way all possible worlds would be put in an order of value"*  
(1931, 176)

## Step 2: Constructing des

### Intermediate goal

A sufficiently rich desirability function to make comparisons of difference in desirability between two worlds meaningful.

↪ define a relation of equal difference of desirability between worlds, in terms of preferences over conditional prospects.

### Problem

Preferences over conditional prospects are driven by both beliefs and desires - we want the relation of equal difference to reflect a difference only in desirability, thus Ramsey needs some way of factoring beliefs out of the picture.

### Definition (Ethically Neutral proposition)

A proposition  $P$  is ethically neutral for agent  $A$  iff  $A$ 's preferences are such that for any  $n$  the following holds:  $\omega_n P \sim \omega_n \neg P$ .

## Step 2: Constructing *des*

### Ethically neutral proposition in action

Suppose that we have:  $\omega_1 \not\sim \omega_2$  and that  $(\omega_1, P; \omega_2) \sim (\omega_2, P; \omega_1)$ , where  $P$  is an ethically neutral proposition. We then infer that:

$$\begin{aligned} des(\omega_1 P) bel(P) + des(\omega_2 \neg P)(1 - bel(P)) = \\ des(\omega_2 P) bel(P) + des(\omega_1 \neg P)(1 - bel(P)) \end{aligned}$$

As  $P$  is an ethically neutral proposition, this simplifies to the following:

$$des(\omega_1) \cdot (2 \cdot bel(P) - 1) = des(\omega_2) \cdot (2 \cdot bel(P) - 1)$$

Since  $\omega_1 \not\sim \omega_2$  this implies:

$$2 \cdot bel(P) - 1 = 0.$$

Thus  $bel(P) = \frac{1}{2}$ . So  $P$  is an ethically neutral proposition of probability  $\frac{1}{2}$ .



## Step 2: Constructing *des*

### Definition (The equal difference relation)

The equal difference relation (denoted by  $=^d$ ) is such that:  
 $(\omega_1, \omega_2) =^d (\omega_3, \omega_4)$  iff  $(\omega_1, P; \omega_4) \sim (\omega_2, P; \omega_3)$ ; where  $P$  is an ethically neutral proposition of probability  $1/2$ .

To see why consider that from  $(\omega_1, P; \omega_4) \sim (\omega_2, P; \omega_3)$  we can infer:

$$des(\omega_1)bel(P) + des(\omega_4)(1 - bel(P)) = des(\omega_2)bel(P) + des(\omega_3)(1 - bel(P))$$

Since  $bel(P) = 1/2$ , we can simplify to obtain:

$$des(\omega_1) - des(\omega_2) = des(\omega_3) - des(\omega_4).$$

# Constructing *des*, determining *bel*

## Constructing *des*

Having defined the equal difference relation Ramsey states 8 conditions on preferences that, according to him, are sufficient to represent an agent as having a desirability function, unique up to a positive linear transformation, such that:  $(\omega_1, \omega_2) =^d (\omega_3, \omega_4)$  iff  $(\omega_1, P; \omega_4) \sim (\omega_2, P; \omega_3)$ , where  $P$  is an ethically neutral proposition of probability  $\frac{1}{2}$ .

## Determining *bel*

He then defines a belief function *bel*. Given that  $\omega_3 \sim (\omega_1, P; \omega_2)$  and that  $\omega_1 \not\sim \omega_2$ :

$$bel(P) =_{def} \frac{des(\omega_3) - des(\omega_2)}{des(\omega_1) - des(\omega_2)}$$

As defined here, *bel* is unique. The reason being that *des* is unique up to a positive linear transformation and thus differences are preserved over those transformations.

# Evaluating the method: assumption of SEU

## Two problems

- 1 SEU isn't true of real agents.
- 2 Ramsey presupposes what he is trying to demonstrate.

## Retort

- 1 Ramsey only thinks it a useful approximation. Pool player analogy.
- 2 Recall two roles for representation theorems. Insofar as it's a charge against him *justifying* a particular view of what constrains rational degrees of belief, it is a weakness. But insofar as we are evaluating his *measurement* of degrees of belief, it simply can't proceed without some background theory. The data needs to fall on some background presumption that then allows to piece it together into a coherent whole.

# Evaluating the method: preference elicitation

## Two problems

- 1 Worlds are too fine-grained to be entertained by real agents.
- 2 The measurement method seems to suffer the problem of significantly altering the measurand (Jeffrey 1983).

## Retort

- 1 As a way out of the problem we simplify Ramsey's ontology and instead of worlds, we deal with propositions (Bradley 2004) and conditional prospects composed of propositions.
- 2 Not so extravagant method. Just ask the person to choose between worlds as-if they could come about (Bradley 2004).

# Evaluating the method: ethically neutral propositions

## Problem

$P$  is an ethically neutral proposition iff agent's preferences are such that  $\omega_n P \sim \omega_n \neg P$  for any  $n$ . For any proposition, there exists some world, where the truth of that proposition is an object of a bet (Sobel 1998).

## Retort

Replace worlds for propositions:  $P$  is an ethically neutral proposition iff for all propositions  $Q$ ,  $PQ \sim \neg PQ$ .

## Retort to the retort

Candidate  $P$ , 'the coin will land heads'. Let  $Q$  be "the coin will land heads and I become a millionaire or the coin will not land heads and I will get into a crippling debt".  $PQ$  is logically equivalent to "coin will land heads and I become a millionaire".  $\neg PQ$  is logically equivalent to "the coin will not land heads and I get into a crippling debt".

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