Universalism in Tarski's Wahrheitsbegriff

1933 and 1935

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Wahrheitsbegriff (1933)

Polish: Pojęcie prawdy w językach nauk dedukcyjnych
(“The Concept of Truth in Formalized Languages”)
Written in 1931.

Aim:

> to construct—with reference to a given language—a materially adequate and formally correct definition of the term ’true sentence’.

Wahrheitsbegriff (1935)

German translation:
Der Wahrheitsbegriff in den formalisierten Sprachen

> it is provided with a Postscript in which some views which had been stated in the Polish original underwent a rather essential revision and modification. (Tarski, 1956)

Which views?
What "essential revision and modification"?

Claims

In the Postscript

1. Tarski abjures (simple) type theory and converts to set theory (Sundholm, 2003);
2. Tarski changes from a universalistic to an anti-universalistic standpoint (De Rouilhan, 1998).

My claim:

1. Modification consists of discarding Leśniewski's Theory of Semantical Categories;
2. Gradual change from Leśniewski's theory to simple type theory;
3. Not clearly anti-universalistic.

Natural language in WB

> ‘it is snowing’ is a true sentence if and only if it is snowing.

Liar paradox:

> The second sentence on this slide is not a true sentence.

Thus:

> the very possibility of a consistent use of the expression ‘true sentence’ which is in harmony with the laws of logic and the spirit of everyday language seems to be very questionable.

Formalized Languages in WB

Artificially constructed languages in which the sense of every expression is unambiguously determined by its form.

Object language: Language about which we speak.

Metalanguage: Language in which we speak.

Convention T

Many different formalized languages. Need for classification.

Theory of Semantical Categories in WB

Every expression belongs to a semantical category.

Principle I:

Two expressions α_1, α_2 belong to the same semantical category, if there exists a propositional function which contains α_1 and which remains a function when α_1 is replaced by α_2.

Principle II:

Two primitive propositional function symbols $P(x_1, \ldots, x_n)$, $Q(y_1, \ldots, y_m)$ belong to the same category if and only if $n = m$, and x_i and y_i belong to the same semantical category for all $i \leq n$.

Classification of categories

Order of expressions:

1. names of individuals and the variables representing them are of the 1st order;

2. a primitive function symbol $P(x_1, \ldots, x_m)$ is of order $n+1$, if the order of $x_i \leq n$ (for all $i \leq m$) and there exists $j \leq m$ s.t. order of $x_j = n$.

Expressions belonging to the same semantical category have the same order, so we can speak of the order of a category.

Classification of languages

1. languages in which all the variables belong to one and the same semantical category;

2. languages in which the number of categories in which the variables are included is greater than 1 but finite;

3. languages in which the variables belong to infinitely many different categories but the order of these variables does not exceed a previously given natural number n;

4. languages which contain variables of arbitrarily high order.

Conclusions of WB

A For every formalized language of finite order a formally correct and materially adequate definition of true sentence can be constructed in the metalanguage, ...

B For formalised languages of infinite order the construction of such a definition is impossible.

C Even with respect to formalized languages of infinite order, the consistent and correct use of the concept of truth is rendered possible by including this concept in the system of primitive concepts of the metalanguage and determining its fundamental properties by means of the axiomatic method.

The role of semantical categories

ϕ is a true sentence if and only if $\phi \in S$ and every infinite sequence of appropriate objects satisfies ϕ.

$(\alpha_1, \alpha_2, \ldots)$ satisfies ϕ if and only if ...

Heterogeneous infinite sequences are impossible.

Instead of "$(\alpha_1, \alpha_2, A_3, \alpha_4, A_5, \ldots)$ satisfies ϕ" we can define

$$(\alpha_1, \alpha_2, \ldots)(A_1, A_2, \ldots) \text{ satisfies } \phi$$

or

$$(\mathfrak{A}_1, \mathfrak{A}_2, \ldots) \text{ satisfies } \phi$$

Conclusions of WB

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Universalism in WB

Universalism:
the belief in the existence of a universal, formal language.

> A universal language is a language of a complete system of logic: a language that contains—actually or potentially—all possible semantical categories which occur in the languages of the deductive sciences.
>
> A universal, formalised language is a language of which all other formalised languages are either fragments, or can be obtained from it or from its fragments by adding certain constants, provided that the semantical categories of these constants are already presented by certain expressions of the given language.

Changes of PS

> [I]t now seems to me interesting and important to inquire what the consequences would be for the basic problems of the present work if we included in the field under consideration formalized languages for which the fundamental principles of the theory of semantical categories no longer hold. (Tarski, PS)

> In order to classify the signs of infinite order (...) (Tarski, PS)

> It does not at all follow from these stipulations that every variable in the languages in question is of a definite order. (Tarski, PS)

What kind of languages are these?

Classification of categories in WB

Order of expressions:

1. names of individuals and the variables representing them are of the 1st order;

2. a primitive function symbol $P(x_1, \ldots, x_m)$ is of order $n+1$, if the order of $x_i \leq n$ (for all $i \leq m$) and there exists $j \leq m$ s.t. order of $x_j = n$.

Expressions belonging to the same semantical category have the same order, so we can speak of the order of a category.

Classification of languages in PS

Order of expressions:

1. names of individuals and the variables representing them are of the 0th order;

2. the order of a particular sign is the smallest ordinal number which is greater than the orders of all arguments in all occurrences of that sign.

The order of a language is the smallest ordinal number which exceeds the orders of all variables occurring in this language.

Conclusions of PS

A* For every formalized language a formally correct and materially adequate definition of true sentence can be constructed in the metalanguage (...)—but under the condition that the metalanguage possesses a higher order than the object language of investigation.

B* If the order of the metalanguage is at most equal to that of the language itself, such a definition cannot be constructed.

[I]t is always possible to construct the metalanguage in such a way that it contains variables of higher order than all the variables of the language studied. The metalanguage then becomes a language of higher order (...)

Syntactic vs Semantic Order

Perhaps set theory?
But can be formalized in a first-order theory.
No variables of indefinite or expressions of transfinite order!

Semantic order:

1. Individuals, i.e. objects which are not sets, are objects of order 0;

2. the order of an arbitrary set is the smallest ordinal number which is greater than the orders of all elements of the sets.

Generalized Universalism

A universal language is a language of a complete system of logic: a language that contains—actually or potentially—all possible semantical categories which occur in the languages of the deductive sciences.

Necessary condition:

If a language is a universal language, then it contains—actually or potentially—all possible orders which occur in the languages of the deductive sciences.

Anti-universalism in PS?

De Rouilhan, 1998:

Suppose that there is a universal language, say U.
Then (PS, A*) a definition of true sentence for U can be given in the metalanguage for U.
This metalanguage is of higher order than U.
So, U does not contain all possible orders and U is not a universal language.
Contradiction.

Moreover: Anti-universalism is at odds with type theory.

Field's Solution

(Field, 2008)

- A^* For every formalized language of which the order of the variables is bound by some ordinal a formally correct and materially adequate definition of true sentence can be constructed in the metalanguage (. . .)—but under the condition that the metalanguage possesses a higher order than the object language of investigation.
- $\bar{B}$ For formalised languages of which the order of the variables is unbound by any ordinal number the construction of a formally correct and materially adequate definition of true sentence is impossible.
- B^* If the order of the metalanguage is at most equal to that of the language itself, such a definition cannot be constructed.

Field's Solution

> [I]t is always possible for languages of bounded order to construct the metalanguage in such a way that it contains variables of higher order than all the variables of the language studied.

Variables of Indefinite and Transfinite Order

(Patterson, 2012)

> In [Princ. Math.] Russell has used the symbol '$\subset$' and many others with arguments of any (finite) level whatever, so that, according to our definition, they belong to the level ω. Russell does not, however, attribute a transfinite level to these, but interprets their mode of use as "systematic ambiguity". (Carnap, 1934)

$$\alpha = \beta$$

$$\forall \alpha . \alpha = \alpha$$

Also Hilbert (1926), Gödel (1931).

Conclusions

1. Modification consists of discarding Leśniewski's Theory of Semantical Categories;

2. Gradual change from Leśniewski's theory to simple type theory;

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