On the simplicity of truth

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Background

An important, non-trivial project:

Produce a satisfactory formal theory of truth.

An important, non-trivial question:

What ought a formal theory of truth look like?

- Invites us to identify a list of **norms** for truth.
- Many good reasons for seeking to identify the norms characteristic of our truth theories.
- Among these: provide a basis for comparing & ranking rival truth theories.

A few examples, to make this a little more concrete:

- The truth predicate ought to behave classically.
- The truth theory ought to have standard models.
- The inner and outer logic of the theory ought to coincide.

Enter simplicity

An idea that has been bouncing around in the literature:

- Truth is simple; and (so) our theory should reflect this fact.

This looks like a good candidate for a truth-theoretic norm.

But: what kind of norm?

- Our truth theory ought to be a theory of simple truth.
- Our truth theory ought to be a simple theory.

Where should the burden of simplicity lie: truth or theory?

(And is there in fact a clear difference?)

If we want to make any progress we need a better grasp of the notion of simplicity, and the normative principle that attaches to it.

Outline of the talk

- What we have

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- What we have
- What we want, but don't (yet) have

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Warning: This is soon-to-be work-in-progress.

So a certain amount of hand-waving is still involved at this stage.

Forking paths

How to understand the demand for simplicity?

(A) As a pre-formal, philosophical requirement.
Then we might e.g. conjecture: simple theories build on simple stories – about how we learn the concept of truth, how we accumulate new truths and revise old ones, etc.
But this is still desperately vague.

Surprising fact 1: The philosophical side of the inquiry is largely under-developed.

(B) As a formal requirement.
Then we should expect independent (possibly different) answers for (i) semantic theories and (ii) axiomatic theories.

Path (Bi) has been explored (in Burgess 1986, Welch 2001).

Surprising fact 2: Path (Bii) has been largely ignored.

Motivations (I)

Sheard 2002 compiles a list of six ‘naïve criteria’ for truth.

Sheard’s third criterion:

> *Simplicity is to be preferred over complex logical or mathematical constructions, however powerful or elegant the latter may be.*
> In most cases in ordinary discourse, the notion of truth is employed to simplify communication. Even in situations in which truth is employed for reasons other than explicit simplification, it is still intended to convey information with an immediacy which disallows the possibility of elaborate superstructures. (Sheard 2002: 177)

Motivations (II)

Halbach & Horsten (2006, forth.) also formulate some desiderata for (deflationist) truth.

Again, simplicity is one of them.

As an explicit desideratum:

> [A truth theory] must satisfy a requirement of naturalness and simplicity. It must contain as few ad hoc elements as possible. (H&H 2006: 207)

As a derivative desideratum:

> [Simplicity] follows from our desideratum of presenting a picture [underpinning the truth theory] in a clear and transparent manner. [...] Simplicity should certainly not be equated with proof-theoretic weakness and, in particular, not with conservativity. (H&H forth.: 11)

Simplicity interpreted: Models (I)

Truth according to Kripke 1975 (Strong Kleene version):

$\mathcal{M}_0 = \langle E_0, A_0 \rangle = \langle \emptyset, \emptyset \rangle$

Successor stages:

$E_{\alpha+1} = \{\phi \in \mathcal{L}_{Tr} : \mathcal{M}_\alpha \models_{SK} \phi\}$

Limit stages (for λ limit ordinal):

$E_\lambda = \bigcup_{\kappa<\lambda} E_\kappa$

<u>Theorem</u> (Minimal fixed-point) There is an ordinal θ that is least and such that $\langle E_\theta, A_\theta \rangle = \langle E_{\theta+1}, A_{\theta+1} \rangle$.

- In the minimal f.p. we *almost* get the unrestricted T-scheme:

$$\textit{For all } \phi \in \mathcal{L}_{Tr} : \mathcal{M}_\theta \models \phi \Leftrightarrow \mathcal{M}_\theta \models Tr(\phi).$$

<u>Theorem</u> (Kripke) The Kripke-truths form a Π^1_1-complete set.

Main relevant fact:

- Π^1_1 complexity is a *relatively low* level of complexity.

Simplicity interpreted: Models (II)

Truth according to Gupta&Belnap 1993:

$$\mathcal{M}_0 = \langle \mathbb{N}, \emptyset \rangle$$

Successor stages:

$$\mathcal{M}_{\alpha+1} = \langle \mathbb{N}, \{\phi \in \mathcal{L}_{Tr} : \mathcal{M}_\alpha \models \phi\} \rangle$$

Limit stages:

$$\mathcal{M}_\lambda = \langle \mathbb{N}, \{\phi \in \mathcal{L}_{Tr} : \exists\beta\forall\gamma(\beta \leqslant \gamma \wedge \gamma < \lambda) \Rightarrow \mathcal{M}_\gamma \models \phi\} \rangle$$

<u>Theorem</u> (Welch) The revision-truths form a Π^1_2-complete set.

Main relevant fact:

- The revision truth sets are much more complicated than the Kripke truth sets.

? $\Rightarrow$ Truth according to the Kripkean theory is simple; truth according to the revision theory is not (or much less so).

Simplicity interpreted: Axioms (I)

The *KF* axioms capture a notion of non-classical truth:

KF_0 PA^{Tr}

KF_1 $\forall \phi_0 \in \mathcal{L}_{PA} : Tr(\phi_0) \leftrightarrow Val(\phi_0) = 1$

KF_2 $\forall \phi_0 \in \mathcal{L}_{PA} : Tr(\neg\phi_0) \leftrightarrow Val(\phi_0) = 0$

KF_3 $\forall \phi \in \mathcal{L}_{Tr} : Tr(Tr(\phi)) \leftrightarrow Tr(\phi)$

KF_4 $\forall \phi \in \mathcal{L}_{Tr} : Tr(\neg Tr(\phi)) \leftrightarrow Tr(\neg\phi)$

KF_5 $\forall \phi \in \mathcal{L}_{Tr} : Tr(\neg\neg\phi) \leftrightarrow Tr(\phi)$

KF_6 $\forall \phi, \psi \in \mathcal{L}_{Tr} : Tr(\phi \wedge \psi) \leftrightarrow (Tr(\phi) \wedge Tr(\psi))$

KF_7 $\forall \phi, \psi \in \mathcal{L}_{Tr} : Tr(\neg(\phi \wedge \psi)) \leftrightarrow (Tr(\neg\phi) \vee Tr(\neg\psi))$

KF_8 $\forall \phi(x) \in \mathcal{L}_{Tr} : Tr(\forall x\, \phi(x)) \leftrightarrow \forall x Tr(\phi(x))$

KF_9 $\forall \phi(x) \in \mathcal{L}_{Tr} : Tr(\neg\forall x\, \phi(x)) \leftrightarrow \neg\forall x Tr(\phi(x))$

KF_{10} $\forall \phi \in \mathcal{L}_{Tr} : \neg\,(Tr(\phi) \wedge Tr(\neg\phi))$

Simplicity interpreted: Axioms (II)

FS captures a classical, compositional notion of truth:

FS_0	PA^{Tr}
FS_1	$\forall \phi_0 \in \mathcal{L}_{PA} : Tr(\phi) \leftrightarrow Val(\phi) = 1$
FS_2	$\forall \phi \in \mathcal{L}_{Tr} : Tr(\neg\phi) \leftrightarrow \neg Tr(\phi)$
FS_3	$\forall \phi, \psi \in \mathcal{L}_{Tr} : Tr(\phi \wedge \psi) \leftrightarrow (Tr(\phi) \wedge Tr(\psi))$
FS_4	$\forall \phi(x) \in \mathcal{L}_{Tr} : Tr(\forall x \phi(x)) \leftrightarrow \forall t Tr(\phi(t/x))$

Nec	From a proof of ϕ, infer $Tr(\phi)$
CoNec	From a proof of $Tr(\phi)$, infer ϕ

Simplicity interpreted: Axioms (III)

In the axiomatic case, it is much less straightforward to see what could count as a measure of simplicity.

Bad idea 1: The simplicity of an axiomatic truth theory is measured by its proof-theoretic strength.

- We would then be led to prefer the weakest possible theories over highly expressive theories.

Bad idea 2: The simplicity of an axiomatic truth theory is measured by the simplicity of its models.

- This seems to miss something: the axiomatic approach is legitimate in its own right, and should have its own canons.

Unhelpful idea: The simplicity of an axiomatic theory lies in the (mathematical) content of the axioms themselves.

- Too vague to be of use...

Theoretical virtues (I)

Taking stock:

- None of the more obvious proposals seems at all promising for settling (Bii) in a satisfactory way.
- We need new leads on how to better understand simplicity.

A natural place to look for ideas: the debate on theoretical virtues in philosophy of science.

- Simplicity is one of several standards by which to evaluate a theory against its rivals.
- This is essentially the same purpose for which we wanted to talk of truth-theoretic desiderata in the first place.
- Note: even in this debate, the issue of simplicity has been treated in piecemeal fashion.
  (So we shouldn't expect to settle the matter in one sitting.)

Theoretical virtues (II)

Discussions of simplicity revolve around three questions:

1. How should simplicity be *defined* (and then *measured*)?
2. What is the *justification* for regarding simplicity to be a virtue?
3. How is simplicity to be *traded-off*?

Elliott Sober on the trade-off question:

> [If] two theories are equally good in all other respects, then the simpler of the two should be regarded as more plausible. However, theories are almost never equally good in all other respects. [...] This means that [...] one needs to establish how sacrifices in simplicity are to be "traded off" against gains in the other factors that affect a theory's plausibility. (2002: 11)

It seems natural to start from Question 1, at least for our purposes.

Theoretical virtues (III)

Sober 2002: "To strive for simplicity in one's theories means that one aims to minimise something."

Three interpretations of simplicity (Baker 2003, 2010):

(a) Syntactic simplicity, or

Elegance: $\approx$ number & complexity of hypotheses

(b) Ontological simplicity, i.e. Occam's Razor or

Parsimony: $\approx$ number & complexity of things postulated

(bi) Quantitative parsimony: $\approx$ number of individual things postulated

(bii) Qualitative parsimony: $\approx$ number of kinds of things postulated

Often indeterminate which of these is being put to work – and why.

Theoretical virtues (IV)

- Philosophers of science who write about theoretical virtues typically focus on elegance and qualitative parsimony.
- Quantitative parsimony often influences formulations of hypotheses, yet is rarely justified.

E.g. Lewis is a sceptic of quantitative parsimony (as we'd expect!):

> I subscribe to the general view that qualitative parsimony is good in a philosophical or empirical hypothesis; but I recognise no presumption whatever in favour of quantitative parsimony. (1973: 87)

- It is crucial to keep elegance and parsimony apart: for often they will pull in opposite directions. (This is the trade-off question.)
- But in order to do so one must have pinned down each kind of simplicity in a precise way. (This is the definition question.)

Simple truth? (I)

What happens if we import this model to the truth case?

(a) **Elegance**: $\approx$ number & complexity of axioms & rules?

$\rightarrow$ *FS* contains fewer axioms than *KF*

$\rightarrow$ Compositionality is built into *FS* cleanly, naturally

$\rightarrow$ *KF* is more cumbersome, fragmented: we need twice as many axioms, and only get positive compositionality in return

$\rightarrow$ On these grounds, *FS* seems to emerge as a clear winner

$\rightarrow$ On this interpretation, simplicity would be a feature of the syntactic identity of our truth theories.

Worry: On the one hand, *FS* has fewer axioms than *KF*; on the other, *FS* contains two *schematic rules* – so really *FS* (vs. *KF*) 'contains' infinitely many sentences...

(A similar worry applies e.g. to the disquotational theory *DT*.)

Simple truth? (II)

(bi) **Quantitative parsimony**: $\approx$ ...

What it shouldn't be:

$\rightarrow$ 'number of things postulated' $\neq$ number of true sentences

$\rightarrow$ 'number of things postulated' $\neq$ number of truth predicates

Other possibilities:

$\rightarrow$ 'number of things postulated' $=$ number of truth values?

$\rightarrow$ 'number of things postulated' $=$ number of principles?

But: worry from previous slide applies again to latter proposal.

(bii) **Qualitative parsimony**: $\approx$ ...

$\rightarrow$ 'number of kinds of things postulated' $=$ number of *kinds*, or *types*, of principles?

This would be a nice way to avoid our recurring worry (above).

Sober 2002: "The problem is to figure out what to count."

Simple truth? (III)

Concrete proposals for meeting the definition challenge, so far:

- The simplicity of a semantic truth theory is measured by the computational complexity of its models.
- The simplicity of an axiomatic truth theory is measured by the number of kinds of principles it encompasses.

It seems we're still missing something...

... possibly because we have yet to address the other challenges:

- Is there a justification for regarding simplicity to be a virtue?
- In science, elegance bring pragmatic advantages: it makes the theory more perspicuous, easier to manipulate, etc.
- Could similar considerations apply to a truth theory?
- We have yet to properly address our first question: is simplicity primarily a feature of truth, or of our truth theories?
- These are just a few of the many open questions on this issue.

The road ahead

What we have:

- A strong intuition: Truth is simple.
- An incomplete philosophical story, mostly based on loose considerations about learning, acquisition of concepts, etc.
- An under-developed formal analysis of simplicity.

What we still need:

- Improve the philosophical story.
- Possible lead: draw on discussions of simplicity as a theoretical virtue in philosophy of science.
- Answer the definition, justification and trade-off questions.
- Bridge the philosophical and the formal analyses of simplicity.

Thank you.

References

- Baker, A. ‘Quantitative parsimony and explanatory power’, *BJPS* 54: 245–259, 2003.
- Baker, A. ‘Simplicity’, *Stanford Encyclopedia of Philosophy*, 2010.
- Belnap, N. and Gupta, N. *The Revision Theory of Truth*, MIT Press, Cambridge, 1993.
- Burgess, J.P. ‘The truth is never simple’, *JSL*, 51(3): 663–681, 1986.
- Halbach, V. and Horsten, L. ‘The deflationist’s axioms for truth’. In *Deflationism and Paradox*, OUP, 2005.
- Halbach, V. & Horsten, L. ‘Norms for theories of reflexive truth’, forth.
- Kripke, S. A. ‘Outline of a theory of truth’, *Journal of Philosophy*, 72(19): 690–716, 1975.
- Lewis, D. *Counterfactuals*, Blackwell, 1973.
- Sheard, M. ‘Truth, provability and naïve criteria’, in Halbach& Horsten (eds.) *Principles of Truth*, Hansel-Hohenhausen, 2002.
- Sober, E. ‘What is the problem of simplicity?’, reprint from Kuzenkamp, McAleer, Zellner (eds.), *Simplicity, inference, and econometric modelling*, CUP 2002.
- Welch, P.W. ‘On Gupta-Belnap revision theories of truth, Kripkean fixed points, and the next stable set’, *Bulletin of Symbolic Logic (Communication)*, 7(3): 345–360, 2001.