On the Strict Tolerant Conception of Truth

Stefan Wintein

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Introduction

What is the Strict Tolerant Conception of Truth?

Bilateralism
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STCT: a novel philosophical and logical approach to truth and semantic paradox, advocated by D. Ripley [PaFoC, forthcoming].
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- Being an inferentialist account, STCT needs a syntactic characterization of its preferred consequence relation.
Ripley presents $\textbf{ST}$: 2 sided sequent calculus for classical logic with truth rules added:

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\Gamma, \sigma \vdash_{\text{ST}} \Delta \\
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**ST** doesn’t have the Cut rule, but so Ripley argues, this rule (*pace* Restall) ”does not follow from the nature of assertion and denial".
A problem for the Strict Tolerant Conception of Truth?

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- What to say about a Liar sentence \( \neg T(\lambda) \)?
- \( \neg T(\lambda) \vdash_{ST} \emptyset \): it is out of bounds to assert the Liar.
- \( \emptyset \vdash_{ST} \neg T(\lambda) \): the Liar is a theorem.
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STCT relies on a distinction between strict and tolerant assertions and denials. Isn’t this distinction all too costly? Ripley: No!

[The strict-tolerant distinction] is not a primitive distinction: we can understand tolerant assertion and denial in terms of their strict cousins, as I’ve presented them here, or we can equally well understand strict in terms of tolerant. So long as we have a grip on one, there is no difficulty in coming to understand the other.
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First, I will develop two formal systems which (jointly) do so.

The strict-tolerant calculus: a \((A^s, D^s, A^t, D^t)\) signed tableau calculus which is sound and complete w.r.t. all four Strong Kleene fixed point consequence relations (including STCT’s favorite one).

Assertoric semantics: a ”semantic version” of the strict-tolerant calculus. Which sentences are actually strictly/tolerantly assertible and/or deniable?
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Second, I will argue that, taken jointly, the systems suggest that Ripley’s claim that the strict-tolerant distinction is not a primitive distinction, has to be reconsidered.
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- Strong Kleene fixed point valuations: \( \{0, \frac{1}{2}, 1\} \) as range.
Let $L_T$ be a f.o. language with truth predicate '$T$'.
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A **ground model** $M = (D, I)$ for $L_T$ is a classical model of $L$ s.t:

$$Sen(L_T) \subseteq D, \quad I([\sigma]) = \sigma$$
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$V_M : Sen(L_T) \to \{0, \frac{1}{2}, 1\}$ is a $SK$ fixed point (over $M$) iff:

- Connectives and quantifiers have a Strong Kleene semantics.

- $V_M$ respects the ground model:
  $$\forall \sigma \in Sen(L) : V_M(\sigma) = C_M(\sigma)$$

- $V_M$ satisfies the identity of truth:
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For simplicity: $R \subseteq Con(L_T)$ s.t. for any $M, M'$:

$\forall r \in R : I(r) = I'(r) \in Sen(L_T)$
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- $FP_M$: all $SK$ fixed points over $M$. 

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  - $\models^{ss}$: All $\alpha \in \Gamma$ valuated as $1 \Rightarrow$ some $\beta \in \Delta$ valuated as $1$.

  - $\models^{tt}$: All $\alpha \in \Gamma$ valuated in $\{\frac{1}{2}, 1\} \Rightarrow$ some $\beta \in \Delta$ valuated in $\{\frac{1}{2}, 1\}$

  - $\models^{st}$: All $\alpha \in \Gamma$ valuated as $1 \Rightarrow$ some $\beta \in \Delta$ valuated in $\{\frac{1}{2}, 1\}$

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- However, (Ripley, Cobreros et al.) non-transitivity is well-located: paradoxical sentences need to be involved.
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- However, (Ripley, Cobreros et al.) non-transitivity is well-located: paradoxical sentences need to be involved.

- Moreover, $\models^{st}$ does preserve a lot of classical meta-inferences and "all failures of classical meta-inferences can be traced down to failures of transitivity".
### Assertoric sentences and fixed point satisfiability

**Assertoric sentence**: sentence of $L_T$ signed with $A^s$, $A^t$, $D^s$ or $D^t$.  

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- **Assertoric sentence**: sentence of $L_T$ signed with $A^s$, $A^t$, $D^s$ or $D^t$.

- A set of assertoric sentences $S$ is **fixed point satisfiable** iff, for some $V$:

  $$A^s_\phi \in S \Rightarrow V(\phi) = 1,$$
  $$D^s_\phi \in S \Rightarrow V(\phi) = 0,$$
  $$A^t_\phi \in S \Rightarrow V(\phi) \in \{1, \frac{1}{2}\},$$
  $$D^t_\phi \in S \Rightarrow V(\phi) \in \{0, \frac{1}{2}\}.$$
**Assertoric sentences and fixed point satisfiability**

- **Assertoric sentence**: sentence of $L_T$ signed with $A^s$, $A^t$, $D^s$ or $D^t$.

- A set of assertoric sentences $S$ is **fixed point satisfiable** iff, for some $V$:

  \[ A^s_\phi \in S \Rightarrow V(\phi) = 1, \quad D^s_\phi \in S \Rightarrow V(\phi) = 0, \]

  \[ A^t_\phi \in S \Rightarrow V(\phi) \in \{1, \frac{1}{2}\}, \quad D^t_\phi \in S \Rightarrow V(\phi) \in \{0, \frac{1}{2}\}. \]

- Rephrase $SK$ consequence in terms of assertoric sentences:
**Assertoric sentences and fixed point satisfiability**

**Introduction**

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of \(SK\) consequence relations

**Assertoric sentences and fixed point satisfiability**

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- A set of assertoric sentences \(S\) is **fixed point satisfiable** iff, for some \(V\):

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A^s_\phi \in S \Rightarrow V(\phi) = 1, \quad D^s_\phi \in S \Rightarrow V(\phi) = 0,
\]

\[
A^t_\phi \in S \Rightarrow V(\phi) \in \{1, \frac{1}{2}\}, \quad D^t_\phi \in S \Rightarrow V(\phi) \in \{0, \frac{1}{2}\}.
\]

- Rephrase \(SK\) consequence in terms of assertoric sentences:

  - \(\Gamma \models^{ss} \Delta\) iff for each fixed point \(V\):

    All \(\alpha \in \Gamma\) valuated as 1 \(\Rightarrow\) some \(\beta \in \Delta\) valuated as 1.
**Assertoric sentences and fixed point satisfiability**

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- Rephrase $SK$ consequence in terms of assertoric sentences:
  - $\Gamma \models^{ss} \Delta$ iff for each fixed point $V$:
    All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.
  - $\Gamma \models^{ss} \Delta$ iff for no fixed point $V$:
    All $\alpha \in \Gamma$ valuated as 1 and all $\beta \in \Delta$ valuated as not-1.
Assertoric sentences and fixed point satisfiability

- **Assertoric sentence**: sentence of $L_T$ signed with $A^s$, $A^t$, $D^s$ or $D^t$.

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  $$A^t_\phi \in S \Rightarrow V(\phi) \in \{1, \frac{1}{2}\}, \quad D^t_\phi \in S \Rightarrow V(\phi) \in \{0, \frac{1}{2}\}.$$

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    All $\alpha \in \Gamma$ valuated as 1 and all $\beta \in \Delta$ valuated in $\{0, \frac{1}{2}\}$. 
### Assertoric sentences and fixed point satisfiability

- **Assertoric sentence**: sentence of $L_T$ signed with $A^s$, $A^t$, $D^s$ or $D^t$.

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  $A^t_\phi \in S \Rightarrow V(\phi) \in \{1, \frac{1}{2}\}$,  
  $D^t_\phi \in S \Rightarrow V(\phi) \in \{0, \frac{1}{2}\}$.

- Rephrase $SK$ consequence in terms of assertoric sentences:
  
  - $\Gamma \vDash^{ss} \Delta$ iff for each fixed point $V$:
    
    All $\alpha \in \Gamma$ valued as 1 $\Rightarrow$ some $\beta \in \Delta$ valued as 1.

  - $\Gamma \vDash^{ss} \Delta$ iff for no fixed point $V$:
    
    All $\alpha \in \Gamma$ valued as 1 and all $\beta \in \Delta$ valued in $\{0, \frac{1}{2}\}$.

  - $\Gamma \vDash^{ss} \Delta$ iff $A^s(\Gamma) \cup D^t(\Delta)$ is not fixed point satisfiable.
**Assertoric sentences and fixed point satisfiability**

- **Assertoric sentence**: sentence of $L_T$ signed with $A^s$, $A^t$, $D^s$ or $D^t$.

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  \]

  \[
  A^t_\phi \in S \Rightarrow V(\phi) \in \{1, \frac{1}{2}\}, \quad D^t_\phi \in S \Rightarrow V(\phi) \in \{0, \frac{1}{2}\}.
  \]

- Rephrase $SK$ consequence in terms of assertoric sentences:

  - $\Gamma \models_{ss} \Delta$ iff for each fixed point $V$:
    All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

  - $\Gamma \models_{ss} \Delta$ iff for no fixed point $V$:
    All $\alpha \in \Gamma$ valuated as 1 and all $\beta \in \Delta$ valuated in $\{0, \frac{1}{2}\}$.

  - $\Gamma \models_{ss} \Delta$ iff $A^s(\Gamma) \cup D^t(\Delta)$ is not fixed point satisfiable.

  - $\Gamma \models_{st} \Delta$ iff $A^s(\Gamma) \cup D^s(\Delta)$ is not fixed point satisfiable.
The strict-tolerant calculus
Here are the tableau rules of our calculus, where $i \in \{s, t\}$:

\[
\begin{align*}
A_i^{\neg \alpha} & \quad D_i^{\neg \alpha} & \quad A_i^{\alpha \lor \beta} & \quad D_i^{\alpha \lor \beta} & \quad A_i^{\alpha \land \beta} & \quad D_i^{\alpha \land \beta} \\
\hline
D_{\alpha} & \quad A_{\alpha} & \quad A_{\alpha} & \quad D_{\alpha}, D_{\beta} & \quad A_{\alpha}, A_{\beta} & \quad D_{\alpha} | D_{\beta} \\
\hline
A_i^T(\sigma) & \quad D_i^T(\sigma) \\
\hline
A_i^{\forall x \phi(x)} & \quad D_i^{\forall x \phi(x)} & \quad u \text{ fresh} & \quad A_i^{\exists x \phi(x)} & \quad D_i^{\exists x \phi(x)} & \quad u \text{ fresh} \\
\hline
A_{\phi(x/c)} & \quad D_{\phi(x/c)} & \quad A_{\phi(x/u)} & \quad D_{\phi(x/u)} & \quad A_{\phi(x/u)} & \quad D_{\phi(x/c)}
\end{align*}
\]
Here are the tableau rules of our calculus, where $i \in \{s, t\}$:

\[
\begin{array}{cccc}
A^i_{\neg \alpha} & D^i_{\neg \alpha} & A^i_{\alpha \lor \beta} & D^i_{\alpha \lor \beta} \\
D^i_{\alpha} & A^i_{\alpha} & A^i_{\alpha, A^i_{\beta}} & D^i_{\alpha, D^i_{\beta}} \\
A^i_{\alpha \land \beta} & D^i_{\alpha \land \beta} & A^i_{T(\sigma)} & D^i_{T(\sigma)} \\
D^i_{\alpha} & D^i_{\alpha, D^i_{\beta}} & A^i_{\forall x \phi(x)} & D^i_{\forall x \phi(x)} \\
A^i_{\phi(x/c)} & D^i_{\phi(x/u)} & u \text{ fresh} & A^i_{\exists x \phi(x)} & D^i_{\exists x \phi(x)} \\
A^i_{\phi(x/u)} & u \text{ fresh} & D^i_{\phi(x/c)} & D^i_{\phi(x/u)} & u \text{ fresh} \\
\end{array}
\]

The $A^t$ rule is valid: $V(\neg \alpha) \in \{\frac{1}{2}, 1\} \implies V(\alpha) \in \{0, \frac{1}{2}\}$
Here are the tableau rules of our calculus, where \( i \in \{s, t\} \): 

\[
\begin{align*}
A^i_{\neg \alpha} & \quad D^i_{\neg \alpha} & \quad A^i_{\alpha \lor \beta} & \quad D^i_{\alpha \lor \beta} & \quad A^i_{\alpha \land \beta} & \quad D^i_{\alpha \land \beta} \\
D^i_{\alpha} & \quad A^i_{\alpha} & \quad A^i_{\alpha} \mid A^i_{\beta} & \quad D^i_{\alpha}, D^i_{\beta} & \quad A^i_{\alpha}, A^i_{\beta} & \quad D^i_{\alpha} \mid D^i_{\beta} \\
A^i_T(\sigma) & \quad D^i_T(\sigma) & \quad A^i_\sigma & \quad D^i_\sigma
\end{align*}
\]

\[
\begin{align*}
A^i_{\forall x \phi(x)} & \quad D^i_{\forall x \phi(x)} & \quad A^i_{\exists x \phi(x)} & \quad D^i_{\exists x \phi(x)} \\
A^i_{\phi(x/c)} & \quad D^i_{\phi(x/u)} & \quad A^i_{\phi(x/\sigma)} & \quad D^i_{\phi(x/c)}
\end{align*}
\]

The \( A^t \) rule is valid: \( V(\neg \alpha) \in \{\frac{1}{2}, 1\} \Rightarrow V(\alpha) \in \{0, \frac{1}{2}\} \)

The \( D^s \) rule is valid: \( V(\alpha \lor \beta) = 0 \Rightarrow V(\alpha) = 0 \& V(\beta) = 0 \), etc.
Here are the tableau rules of our calculus, where $i \in \{s, t\}$:

\[
\begin{array}{cccccc}
A^i_{\neg \alpha} & D^i_{\neg \alpha} & A^i_{\alpha \lor \beta} & D^i_{\alpha \lor \beta} & A^i_{\alpha \land \beta} & D^i_{\alpha \land \beta} \\
\hline
D^i_{\alpha} & A^i_{\alpha} & D^i_{\alpha \lor \beta} & A^i_{\alpha \land \beta} & D^i_{\alpha \land \beta}
\end{array}
\]

\[
\begin{array}{cc}
A^i_{T(\sigma)} & D^i_{T(\sigma)} \\
\hline
A^i_{\sigma} & D^i_{\sigma}
\end{array}
\]

\[
\begin{array}{cccc}
A^i_{\forall x \phi(x)} & D^i_{\forall x \phi(x)} & A^i_{\exists x \phi(x)} & D^i_{\exists x \phi(x)} \\
\hline
A^i_{\phi(x/c)} & D^i_{\phi(x/c)} & A^i_{\phi(x/u)} & D^i_{\phi(x/u)}
\end{array}
\]

The $A^t_\neg$ rule is valid: $V(\neg \alpha) \in \{\frac{1}{2}, 1\} \Rightarrow V(\alpha) \in \{0, \frac{1}{2}\}$

The $D^s_\lor$ rule is valid: $V(\alpha \lor \beta) = 0 \Rightarrow V(\alpha) = 0 \& V(\beta) = 0$, etc.

Observe: no strict-to-tolerant rules.
A set of assertoric sentences $S$ is closed iff:

- For some sentence $\sigma$ of $L_T$: $\{A^s_\sigma, D^s_\sigma\} \subseteq S$
- For some truth-free sentence $\sigma$ of $L$: $\{A^t_\sigma, D^t_\sigma\} \subseteq S$
- For some sentence $\sigma$ of $L_T$: $\{A^s_\sigma, D^t_\sigma\} \subseteq S$
- For some sentence $\sigma$ of $L_T$: $\{A^t_\sigma, D^s_\sigma\} \subseteq S$
A set of assertoric sentences $S$ is closed iff:

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- For some sentence $\sigma$ of $L_T$: $\{A^s_\sigma, D^t_\sigma\} \subseteq S$
- For some sentence $\sigma$ of $L_T$: $\{A^t_\sigma, D^s_\sigma\} \subseteq S$

When $S$ is closed it is not fixed point satisfiable.
A set of assertoric sentences $S$ is closed iff:

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- For some sentence $\sigma$ of $L_T$: $\{A^s_\sigma, D^t_\sigma\} \subseteq S$
- For some sentence $\sigma$ of $L_T$: $\{A^t_\sigma, D^s_\sigma\} \subseteq S$

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We define 4 syntactic $SK$ consequence relations $\vdash^{ij}$, e.g.: 
A set of assertoric sentences $S$ is closed iff:

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We define 4 syntactic $SK$ consequence relations $\vdash_{ij}$, e.g.:

- $\Gamma \vdash_{ss} \Delta \iff A^s(\Gamma) \cup D^t(\Delta)$ has a closed tableau.
A set of assertoric sentences $S$ is closed iff:

- For some sentence $\sigma$ of $L_T$: $\{A^s_\sigma, D^s_\sigma\} \subseteq S$
- For some truth-free sentence $\sigma$ of $L$: $\{A^t_\sigma, D^t_\sigma\} \subseteq S$
- For some sentence $\sigma$ of $L_T$: $\{A^s_\sigma, D^t_\sigma\} \subseteq S$
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We define 4 syntactic $SK$ consequence relations $\vdash_{ij}$, e.g.:

- $\Gamma \vdash_{ss} \Delta \iff A^s(\Gamma) \cup D^t(\Delta)$ has a closed tableau.
- $\Gamma \vdash_{st} \Delta \iff A^s(\Gamma) \cup D^s(\Delta)$ has a closed tableau.
A set of assertoric sentences \( S \) is closed iff:

- For some sentence \( \sigma \) of \( L_T \): \( \{ A^s_\sigma, D^s_\sigma \} \subseteq S \)
- For some truth-free sentence \( \sigma \) of \( L \): \( \{ A^t_\sigma, D^t_\sigma \} \subseteq S \)
- For some sentence \( \sigma \) of \( L_T \): \( \{ A^s_\sigma, D^t_\sigma \} \subseteq S \)
- For some sentence \( \sigma \) of \( L_T \): \( \{ A^t_\sigma, D^s_\sigma \} \subseteq S \)

When \( S \) is closed it is not fixed point satisfiable.

We define 4 syntactic \( SK \) consequence relations \( \vdash^{ij} \), e.g.:

- \( \Gamma \vdash^{ss} \Delta \iff A^s(\Gamma) \cup D^t(\Delta) \) has a closed tableau.
- \( \Gamma \vdash^{st} \Delta \iff A^s(\Gamma) \cup D^s(\Delta) \) has a closed tableau.

**Theorem**: \( \vdash^{ij} \) is sound and complete w.r.t. \( \models^{ij} \).
Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $V^s_M$ and $V^t_M$

Why the strict-tolerant distinction is primitive
The strict-tolerant calculus characterizes $SK$ consequence in strict-tolerant terms.
The strict-tolerant calculus characterizes \( SK \) consequence in strict-tolerant terms.

But, which sentences are actually strictly/tolerantly assertible/deniable? What about ‘snow is white’? What about the Truthgetter?
The semantic counterpart of the strict-tolerant calculus

- The strict-tolerant calculus characterizes $SK$ consequence in strict-tolerant terms.

- But, which sentences are actually strictly/tolerantly assertible/deniable? What about ‘snow is white’? What about the Truth teller?

- Given a fixed ground model $M$: what is the strict/tolerant assertoric status of $L_T$ sentences?
The semantic counterpart of the strict-tolerant calculus

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- Answer via assertoric semantics (”semantic strict-tolerant calculus”):
The semantic counterpart of the strict-tolerant calculus

\[ \text{The strict-tolerant calculus characterizes } \mathcal{SK} \text{ consequence in strict-tolerant terms.} \]

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\[ \text{Given a fixed ground model } \mathcal{M}: \text{ what is the strict/tolerant assertoric status of } L_T \text{ sentences?} \]

\[ \text{Answer via } \textbf{assertoric semantics} \text{ ("semantic strict-tolerant calculus"):} \]

- Quantifiers now range over the domain of \( \mathcal{M} \).
The strict-tolerant calculus characterizes $SK$ consequence in strict-tolerant terms.

But, which sentences are actually strictly/tolerantly assertible/deniable? What about ‘snow is white’? What about the Truth-teller?

Given a fixed ground model $M$: what is the strict/tolerant assertoric status of $L_T$ sentences?

Answer via assertoric semantics (”semantic strict-tolerant calculus”):

- Quantifiers now range over the domain of $M$.
- Closure conditions of the strict-tolerant calculus are augmented:
  Not allowed to assert (strictly or tolerantly) $\sigma$ of $L$ if $C_M(\sigma) = 0$.
  Not allowed to deny (strictly or tolerantly) $\sigma$ of $L$ if $C_M(\sigma) = 1$. 
Example: strict assertoric trees

Let $\sigma := \neg T(\lambda) \land W(s)$. Is $\sigma$ strictly assertible /deniable?
Example: strict assertoric trees

Let $\sigma := \neg T(\lambda) \land W(s)$. Is $\sigma$ strictly assertible /deniable?

To answer, compute **strict assertoric trees**: $\mathcal{X}^{\sigma}_{A_s}$ and $\mathcal{X}^{\sigma}_{D_s}$:

\begin{align*}
A_s^T(\lambda) \land W(s) & \\
A_s^T(\lambda) & \\
A_s^W(s) & \\
D_s^T(\lambda) & \\
D_s^T(\lambda) & \\
A_s^T(\lambda) & \\

D_s^T(\lambda) & \\
D_s^W(s) & \\
A_s^T(\lambda) & \\
A_s^T(\lambda) & \\
D_s^T(\lambda) & \\

\end{align*}
Example: strict assertoric trees

Let $\sigma := \neg T(\lambda) \land W(s)$. Is $\sigma$ strictly assertible /deniable?

To answer, compute **strict assertoric trees**: $\mathcal{T}_A^\sigma$ and $\mathcal{T}_D^\sigma$:

Both $\mathcal{T}_A^\sigma$ and $\mathcal{T}_D^\sigma$ are closed $M$.
Let $\sigma := \neg T(\lambda) \land W(s)$. Is $\sigma$ strictly assertible /deniable?

To answer, compute **strict assertoric trees**: $\Sigma_{A^s}^{\sigma}$ and $\Sigma_{D^s}^{\sigma}$:

Both $\Sigma_{A^s}^{\sigma}$ and $\Sigma_{D^s}^{\sigma}$ are closed $M$.

$\mathcal{V}_M^s(\sigma) = (0, 0)$: $\sigma$ neither strictly assertible nor deniable.
Example: tolerant assertoric trees

Let $\sigma := \neg T(\lambda) \land W(s)$. Is $\sigma$ tolerantly assertible/deniable?
Let $\sigma := \neg T(\lambda) \land W(s)$. Is $\sigma$ tolerantly assertible /deniable?

To answer, compute tolerant assertoric trees: $\mathcal{A}_A^\sigma$ and $\mathcal{A}_D^\sigma$:

```
\begin{align*}
A^t_{\neg T(\lambda) \land W(s)} & \\
& \quad A^t_{\neg T(\lambda)} \\
& \quad \quad A^t_{W(s)} \\
& \quad \quad \quad D^t_{T(\lambda)} \\
& \quad \quad \quad \quad D^t_{\neg T(\lambda)} \\
& \quad \quad \quad \quad \quad A^t_{T(\lambda)} \\
\end{align*}
```

```
\begin{align*}
D^t_{\neg T(\lambda) \land W(s)} & \\
& \quad D^t_{\neg T(\lambda)} \\
& \quad \quad D^t_{W(s)} \\
& \quad \quad \quad A^t_{T(\lambda)} \\
& \quad \quad \quad \quad D^t_{T(\lambda)} \\
\end{align*}
```
Let $\sigma := \neg T(\lambda) \land W(s)$. Is $\sigma$ tolerantly assertible /deniable?

To answer, compute **tolerant assertoric trees**: $\Sigma^\sigma_A$ and $\Sigma^\sigma_D$:

Both $\Sigma^\sigma_A$ and $\Sigma^\sigma_D$ are open in $\mathcal{M}$. 

$$\begin{align*}
A^t_{\neg T(\lambda) \land W(s)} & \quad D^t_{\neg T(\lambda) \land W(s)} \\
A^t_{\neg T(\lambda)} & \quad D^t_{\neg T(\lambda)} \\
A^t_{W(s)} & \quad D^t_{W(s)} \\
D^t_{T(\lambda)} & \quad A^t_{T(\lambda)} \\
D^t_{T(\lambda)} & \quad A^t_{T(\lambda)} \\
A^t_{T(\lambda)} & \quad D^t_{T(\lambda)}
\end{align*}$$
Let $\sigma := \neg T(\lambda) \land W(s)$. Is $\sigma$ tolerantly assertible/deniable?

To answer, compute tolerant assertoric trees: $\mathcal{Y}_A^\sigma$ and $\mathcal{Y}_D^\sigma$:

Both $\mathcal{Y}_A^\sigma$ and $\mathcal{Y}_D^\sigma$ are open $M$.

$\mathcal{V}_M^t(\sigma) = (1, 1)$: $\sigma$ both tolerantly assertible and deniable.
Inducing familiar valuations

With $M$ a ground model, $V^s_M$ and $V^t_M$ are induced as follows. With $i, j \in \{s, t\}$:

$$V^i_M(\sigma) = \begin{cases} 
(1, 0), & \Sigma^\sigma_A^i \text{ is open}_M & \Sigma^\sigma_D^i \text{ is closed}_M \\
(1, 1), & \Sigma^\sigma_A^i \text{ is open}_M & \Sigma^\sigma_D^i \text{ is open}_M \\
(0, 0), & \Sigma^\sigma_A^i \text{ is closed}_M & \Sigma^\sigma_D^i \text{ is closed}_M \\
(0, 1), & \Sigma^\sigma_A^i \text{ is closed}_M & \Sigma^\sigma_D^i \text{ is open}_M 
\end{cases}$$
Inducing familiar valuations

With $M$ a ground model, $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$ are induced as follows. With $i, j \in \{s, t\}$:

$$\mathcal{V}^i_M(\sigma) = \begin{cases} 
(1, 0), & \mathcal{V}^\sigma_{A^i} \text{ is open}_M \land \mathcal{V}^\sigma_{D^i} \text{ is closed}_M \\
(1, 1), & \mathcal{V}^\sigma_{A^i} \text{ is open}_M \land \mathcal{V}^\sigma_{D^i} \text{ is open}_M \\
(0, 0), & \mathcal{V}^\sigma_{A^i} \text{ is closed}_M \land \mathcal{V}^\sigma_{D^i} \text{ is closed}_M \\
(0, 1), & \mathcal{V}^\sigma_{A^i} \text{ is closed}_M \land \mathcal{V}^\sigma_{D^i} \text{ is open}_M 
\end{cases}$$

**Theorem** $\mathcal{V}^s_M$ is equivalent to Kripke's $\mathcal{K}_M^4$, where:

$$\mathcal{K}_M^4(\sigma) = (1, 0) \iff \exists V_M : V_M(\sigma) = 1 \land \nexists V_M : V_M(\sigma) = 0$$

$$\mathcal{K}_M^4(\sigma) = (1, 1) \iff \exists V_M : V_M(\sigma) = 1 \land \exists V_M : V_M(\sigma) = 0$$

$$\mathcal{K}_M^4(\sigma) = (0, 0) \iff \nexists V_M : V_M(\sigma) = 1 \land \nexists V_M : V_M(\sigma) = 0$$

$$\mathcal{K}_M^4(\sigma) = (0, 1) \iff \nexists V_M : V_M(\sigma) = 1 \land \exists V_M : V_M(\sigma) = 0$$
Inducing familiar valuations

With $M$ a ground model, $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$ are induced as follows. With $i, j \in \{s, t\}$:

$$\mathcal{V}_M^i(\sigma) = \begin{cases} (1, 0), & \mathcal{S}_{A}^\sigma_i \text{ is open}_M & \mathcal{S}_{D}^\sigma_i \text{ is closed}_M \\ (1, 1), & \mathcal{S}_{A}^\sigma_i \text{ is open}_M & \mathcal{S}_{D}^\sigma_i \text{ is open}_M \\ (0, 0), & \mathcal{S}_{A}^\sigma_i \text{ is closed}_M & \mathcal{S}_{D}^\sigma_i \text{ is closed}_M \\ (0, 1), & \mathcal{S}_{A}^\sigma_i \text{ is closed}_M & \mathcal{S}_{D}^\sigma_i \text{ is open}_M \end{cases}$$

**Theorem** $\mathcal{V}_M^s$ is equivalent to Kripke’s $\mathcal{K}_M^4$, where:

- $\mathcal{K}_M^4(\sigma) = (1, 0) \iff \exists V_M : V_M(\sigma) = 1$ and $\not\exists V_M : V_M(\sigma) = 0$
- $\mathcal{K}_M^4(\sigma) = (1, 1) \iff \exists V_M : V_M(\sigma) = 1$ and $\exists V_M : V_M(\sigma) = 0$
- $\mathcal{K}_M^4(\sigma) = (0, 0) \iff \not\exists V_M : V_M(\sigma) = 1$ and $\not\exists V_M : V_M(\sigma) = 0$
- $\mathcal{K}_M^4(\sigma) = (0, 1) \iff \exists V_M : V_M(\sigma) = 1$ and $\not\exists V_M : V_M(\sigma) = 0$

**Theorem** $\mathcal{V}_M^t : \text{Sen}(L_T) \rightarrow \{(1, 0), (1, 1), (0, 1)\}$ is equivalent to the minimal fixed point over $M$. 

\[
\begin{align*}
\mathcal{K}_M^4(\sigma) = (1, 0) & \iff \exists V_M : V_M(\sigma) = 1 \\
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\]
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Some remarks on the interpretation of $V^s_M$ and $V^t_M$

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$\nu^s_M$ and $\nu^t_M$ model *initial* assertoric possibilities.
Performing strict/tolerant assertoric actions rules out other such actions.
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  Performing strict/tolerant assertoric actions rules out other such actions.

- The transmission of assertoric possibilities due to (strict and tolerant) assertions and denials is captured by the strict-tolerant calculus.
Why the strict-tolerant distinction is primitive
$$\text{Why bilateralism?}$$

- We interpret $A^s$, $D^s$, $A^t$ and $D^t$ as force indicators.
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  [The strict-tolerant distinction] it is not a primitive distinction; we can understand tolerant assertion and denial in terms of their strict cousins, as I’ve presented them here, or we can equally well understand strict in terms of tolerant. So long as we have a grip on one, there is no difficulty in coming to understand the other.

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The remark is to the point relative to a particular fixed point $V_M$:

- $\sigma$ is strongly $V_M$ assertible $\iff$ $\sigma$ is not tolerantly $V_M$ deniable
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The strict and tolerant can be understood in terms of one another if, given $M$, there would be a *privileged* $V_M^*$ which would inform us about the assertoric status of the $L_T$ sentences.
Supervenience of Semantics and the Fixed Point Conception

- The strict and tolerant can be understood in terms of one another if, given $M$, there would be a privileged $V^*_M$ which would inform us about the assertoric status of the $L_T$ sentences.

- **Supervenience of semantics**: Once all the empirical facts have been settled, so are all the semantic facts. In terms of our formal theory, the intuition becomes: for any given ground model, there is exactly one correct interpretation of the truth predicate. — M. Kremer 1988
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**Fixed Point Conception of Truth:** This criterion takes the notion of a fixed point to give the whole meaning of true. Or, in Kripke’s words, the intuitive concept of truth is expressed by the formula: ‘we are entitled to assert (or deny) of a sentence that it is true precisely under the circumstances when we can assert (or deny) the sentence itself’. M. Kremer 1988
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- Due to its inferentialist commitments, STCT is committed to **FPCT**.

- Moreover, as $V^s_M$ and $V^t_M$ do not determine each other, we can’t understand the strict in terms of tolerant (nor vice versa).
Thus, we need 4 distinct primitive speech acts?
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Not so fast. For look at assertoric semantics:
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- The strict-tolerant calculus does so in its closure conditions:
  \[ A_s^\sigma, D_t^\sigma \text{ or } A_t^\sigma, D_s^\sigma \text{ occur on a tableau path.} \]
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- The strict-tolerant calculus does so in its closure conditions:
  $A^s_\sigma$, $D^t_\sigma$ or $A^t_\sigma$, $D^s_\sigma$ occur on a tableau path.

- Hence, in order to understand the relations between strict and tolerant actions, it seems that we must understand the signs $A^s$, $D^s$, $A^t$, $D^t$ as primitive force indicators.
- Perhaps then, the strict and tolerant can’t be understood in terms of one another.
But not so fast... Argument 2

- Perhaps then, the strict and tolerant can’t be understood in terms of one another.

- But who cares? What is at the heart of STCT is a syntactic (bilaterlistic) characterization of $\models^{st}$. The strict and tolerant are at least on a par as we can do so by putting constraints either on:
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$\models^{st}$ can be characterized by putting constraints on strict assertions and denials (as in $\models^{st}$) but not in terms their tolerant cousins.

According to the strict-tolerant calculus, strict assertions and denials have a privileged status.
But but not so fast... Argument 3

- But we can easily restore the asymmetry between the strict and tolerant by characterizing $\models^{st}$ via the notion of \textit{refusal}: 

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- But we can easily restore the asymmetry between the strict and tolerant by characterizing $\models^{st}$ via the notion of *refusal*:

- Refusal-to-tolerantly-deny all premisses and refusal-to-tolerantly-assert all consequences is out of bounds
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- But we can easily restore the asymmetry between the strict and tolerant by characterizing $\models_{st}$ via the notion of *refusal*:

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- Then again, even a detailed notion of refusal (and acceptance) only shows that the strict-tolerant symmetry can be ”restored” via auxiliary notions.
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The strict-tolerant calculus and assertoric semantics shed light on STCT and suggest that the strict-tolerant distinction needs further attention.
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The strict-tolerant calculus and assertoric semantics shed light on STCT and suggest that the strict-tolerant distinction needs further attention.

Thanks for your attention.