On the Strict Tolerant Conception of Truth

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Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

Introduction

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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What is the Strict Tolerant Conception of Truth?

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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What is the Strict Tolerant Conception of Truth?

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- Non-transitive consequence relations are studied in several papers (on vagueness and truth) by P.Cobreros, P.Egré, D.Ripley and R. van Rooij.

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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  - *Inferentialism*: meaning is to be explained by correctness of inference.
  - *Bilateralism*: correctness of inference is to be explained by constraints on assertion *and denial*. (e.g. Rumfitt, Restall *pace* Dummett)

What is the Strict Tolerant Conception of Truth?

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- Being an inferentialist account, STCT needs a syntactic characterization of its preferred consequence relation.

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

- Ripley presents **ST**: 2 sided sequent calculus for classical logic with truth rules added:

$$\frac{\Gamma, \sigma \vdash^{\mathbf{ST}} \Delta}{\Gamma, T(\overline{\sigma}) \vdash^{\mathbf{ST}} \Delta} \qquad \frac{\Gamma \vdash^{\mathbf{ST}} \sigma, \Delta}{\Gamma \vdash^{\mathbf{ST}} T(\overline{\sigma}), \Delta}$$

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- STCT's bilateralism is reflected in its sequent interpretation:

  $\Gamma \vdash^{\mathbf{ST}} \Delta$: "out of bounds to assert all of Γ and to deny all of Δ"

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still ...
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- **ST** doesn't have the Cut rule, but so Ripley argues, this rule (*pace* Restall) "does not follow from the nature of assertion and denial".

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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A problem for the Strict Tolerant Conception of Truth?

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still ...
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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A problem for the Strict Tolerant Conception of Truth?

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still ...
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still ...
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- STCT relies on a distinction between strict and tolerant assertions and denials. Isn't this distinction all too costly? Ripley: No!

> [The strict-tolerant distinction] is not a primitive distinction: we can understand tolerant assertion and denial in terms of their strict cousins, as I've presented them here, or we can equally well understand strict in terms of tolerant. So long as we have a grip on one, there is no difficulty in coming to understand the other.

But still . . .

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- First, I will develop two formal systems which (jointly) do so.
- **The strict-tolerant calculus**: a (A^s, D^s, A^t, D^t) signed tableau calculus which is sound and complete w.r.t. all four *Strong Kleene fixed point consequence relations* (including STCT's favorite one).
- **Assertoric semantics**: a "semantic version" of the strict-tolerant calculus. Which sentences are *actually* strictly/tolerantly assertible and/or deniable?

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- **Assertoric semantics**: a "semantic version" of the strict-tolerant calculus. Which sentences are *actually* strictly/tolerantly assertible and/or deniable?
- Second, I will argue that, taken jointly, the systems suggest that Ripley's claim that the strict-tolerant distinction is not a primitive distinction, has to be reconsidered.

Introduction
What is the Strict Tolerant Conception of Truth?
Bilateralism
A problem for the Strict Tolerant Conception of Truth?
But still . . .
Overview
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

From classical to Strong Kleene consequence

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

- Classical logic recognizes two semantic values: 1 and 0.

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

- Classical logic recognizes two semantic values: 1 and 0.
- Which can be used to define classical consequence.

$\Gamma \vDash^{cl} \Delta$ just in case:

- All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- These are all equivalent, as ”non-0 = 1” in classical logic.

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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- But not when there are more than 2 semantic values.

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- *Strong Kleene fixed point valuations*: $\{0, \frac{1}{2}, 1\}$ as range.

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Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

- Let L_T be a f.o. language with truth predicate 'T'.

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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- A **ground model** $M = (D, I)$ for L_T is a classical model of L s.t:

$$Sen(L_T) \subseteq D, \qquad I([\sigma]) = \sigma$$

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- $V_M : Sen(L_T) \to \{0, \frac{1}{2}, 1\}$ is a SK **fixed point** (over M) iff:
  - Connectives and quantifiers have a Strong Kleene semantics.
  - V_M respects the ground model: $\forall \sigma \in Sen(L) : V_M(\sigma) = \mathcal{C}_M(\sigma)$
  - V_M satisfies the identity of truth: $\forall \sigma \in Sen(L_T) : V_M(T(\overline{\sigma})) = V_M(\sigma)$

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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    $\forall \sigma \in Sen(L_T) : V_M(T(\overline{\sigma})) = V_M(\sigma)$
- For simplicity: $\mathcal{R} \subseteq Con(L_T)$ s.t. for any M, M':

$$\forall r \in \mathcal{R} : I(r) = I'(r) \in Sen(L_T)$$

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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  $V_M(\sigma) = 1 \quad (0)$: σ is *strictly* assertible (deniable).
  $V_M(\sigma) \in \{1, \frac{1}{2}\} \quad (\in \{0, \frac{1}{2}\})$: σ is *tolerantly* assertible (deniable).

Strict-tolerant slang and Strong Kleene consequence

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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- $V_M(\sigma) = \frac{1}{2}$: σ is neither strictly, both tolerantly assertible/deniable.

Strict-tolerant slang and Strong Kleene consequence

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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- $V_M(\sigma) = \frac{1}{2}$: σ is neither strictly, both tolerantly assertible/deniable.
- $\mathbf{FP}_M$: all SK fixed points over M.

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Strict-tolerant slang and Strong Kleene consequence

- When V_M is a SK fixed point, we say that:
  $V_M(\sigma) = 1 \quad (0)$: σ is *strictly* assertible (deniable).
  $V_M(\sigma) \in \{1, \frac{1}{2}\} \quad (\in \{0, \frac{1}{2}\})$: σ is *tolerantly* assertible (deniable).
- $V_M(\sigma) = \frac{1}{2}$: σ is neither strictly, both tolerantly assertible/deniable.
- $\mathbf{FP}_M$: all SK fixed points over M.
  $\mathbf{FP}$: $V \in \mathbf{FP} \Leftrightarrow V \in \mathbf{FP}_M$ for some M.

Strict-tolerant slang and Strong Kleene consequence

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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- Quantify over $\mathbf{FP}$ to define 4 SK consequence relations:

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

Strict-tolerant slang and Strong Kleene consequence

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- Quantify over $\mathbf{FP}$ to define 4 SK consequence relations:

  - All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.
  - All $\alpha \in \Gamma$ valuated as non-0 $\Rightarrow$ some $\beta \in \Delta$ valuated as non-0
  - All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as non-0
  - All $\alpha \in \Gamma$ valuated as non-0 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- Quantify over $\mathbf{FP}$ to define 4 SK consequence relations:

$\vDash^{ss}$: All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

$\vDash^{tt}$: All $\alpha \in \Gamma$ valuated in $\{\frac{1}{2}, 1\}$ $\Rightarrow$ some $\beta \in \Delta$ valuated in $\{\frac{1}{2}, 1\}$

$\vDash^{st}$: All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated in $\{\frac{1}{2}, 1\}$

$\vDash^{ts}$: All $\alpha \in \Gamma$ valuated in $\{\frac{1}{2}, 1\}$ $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

Some pros and cons of SK consequence relations

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

- All 4 SK consequence relations: *transparent truth*.

Some pros and cons of SK consequence relations

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- $\models^{ss}$ violates identity: $\not\models^{ss} \alpha \to \alpha$.

Some pros and cons of SK consequence relations

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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Some pros and cons of SK consequence relations

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- Not so for $\models^{st}$: whenever an argument form is classically valid, it is also $\models^{st}$ valid.

Some pros and cons of SK consequence relations

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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- Not so for $\models^{st}$: whenever an argument form is classically valid, it is also $\models^{st}$ valid.
- $\models^{st}$ is non-transitive: $\alpha \models^{st} \beta \;\&\; \beta \models^{st} \gamma \not\Rightarrow \alpha \models^{st} \gamma$.

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- $\models^{st}$ is non-transitive: $\alpha \models^{st} \beta \;\&\; \beta \models^{st} \gamma \not\Rightarrow \alpha \models^{st} \gamma$.
- However, (Ripley, Cobreros et all.) non-transitivity is well-located: paradoxical sentences need to be involved.

Some pros and cons of SK consequence relations

- All 4 SK consequence relations: *transparent truth*.
- $\models^{ss}$ violates identity: $\not\models^{ss} \alpha \rightarrow \alpha$.
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- $\models^{st}$ is non-transitive: $\alpha \models^{st} \beta \;\&\; \beta \models^{st} \gamma \not\Rightarrow \alpha \models^{st} \gamma$.
- However, (Ripley, Cobreros et all.) non-transitivity is well-located: paradoxical sentences need to be involved.
- Moreover, $\models^{st}$ does preserve a lot of classical meta-inferences and ”all failures of classical meta-inferences can be traced down to failures of transitivity”.

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

- *Assertoric sentence*: sentence of L_T signed with A^s, A^t, D^s or D^t.

Assertoric sentences and fixed point satisfiability

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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Assertoric sentences and fixed point satisfiability

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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$$A^s_\phi \in S \Rightarrow V(\phi) = 1, \qquad D^s_\phi \in S \Rightarrow V(\phi) = 0,$$

$$A^t_\phi \in S \Rightarrow V(\phi) \in \{1, \tfrac{1}{2}\}, \qquad D^t_\phi \in S \Rightarrow V(\phi) \in \{0, \tfrac{1}{2}\}.$$

Assertoric sentences and fixed point satisfiability

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- Rephrase SK consequence in terms of assertoric sentences:

Assertoric sentences and fixed point satisfiability

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- Rephrase SK consequence in terms of assertoric sentences:

- $\Gamma \vDash^{ss} \Delta$ iff for each fixed point V:
All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

Assertoric sentences and fixed point satisfiability

Introduction
From classical to Strong Kleene consequence
Classical consequence
Strong Kleene fixed point (valuations)
Strict-tolerant slang and Strong Kleene consequence
Some pros and cons of SK consequence relations
Assertoric sentences and fixed point satisfiability
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive

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All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

- $\Gamma \vDash^{ss} \Delta$ iff for no fixed point V:
All $\alpha \in \Gamma$ valuated as 1 and all $\beta \in \Delta$ valuated as not-1.

Assertoric sentences and fixed point satisfiability

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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Assertoric sentences and fixed point satisfiability

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

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All $\alpha \in \Gamma$ valuated as 1 and all $\beta \in \Delta$ valuated in $\{0, \frac{1}{2}\}$.

- $\Gamma \vDash^{ss} \Delta$ iff $A^s(\Gamma) \cup D^t(\Delta)$ is not fixed point satisfiable.

Assertoric sentences and fixed point satisfiability

Introduction

From classical to Strong Kleene consequence

Classical consequence

Strong Kleene fixed point (valuations)

Strict-tolerant slang and Strong Kleene consequence

Some pros and cons of SK consequence relations

Assertoric sentences and fixed point satisfiability

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

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- Rephrase SK consequence in terms of assertoric sentences:

- $\Gamma \vDash^{ss} \Delta$ iff for each fixed point V:
  All $\alpha \in \Gamma$ valuated as 1 $\Rightarrow$ some $\beta \in \Delta$ valuated as 1.

- $\Gamma \vDash^{ss} \Delta$ iff for no fixed point V:
  All $\alpha \in \Gamma$ valuated as 1 and all $\beta \in \Delta$ valuated in $\{0, \frac{1}{2}\}$.

- $\Gamma \vDash^{ss} \Delta$ iff $A^s(\Gamma) \cup D^t(\Delta)$ is not fixed point satisfiable.

- $\Gamma \vDash^{st} \Delta$ iff $A^s(\Gamma) \cup D^s(\Delta)$ is not fixed point satisfiable.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

The strict-tolerant calculus

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

Tableau rules

Here are the tableau rules of our calculus, where $i \in \{s, t\}$:

$$\frac{A^i_{\neg\alpha}}{D^i_\alpha} \quad \frac{D^i_{\neg\alpha}}{A^i_\alpha} \qquad \frac{A^i_{\alpha\vee\beta}}{A^i_\alpha \mid A^i_\beta} \quad \frac{D^i_{\alpha\vee\beta}}{D^i_\alpha, D^i_\beta} \qquad \frac{A^i_{\alpha\wedge\beta}}{A^i_\alpha, A^i_\beta} \quad \frac{D^i_{\alpha\wedge\beta}}{D^i_\alpha \mid D^i_\beta}$$

$$\frac{A^i_{T(\overline{\sigma})}}{A^i_\sigma} \qquad \frac{D^i_{T(\overline{\sigma})}}{D^i_\sigma}$$

$$\frac{A^i_{\forall x\phi(x)}}{A^i_{\phi(x/c)}} \quad \frac{D^i_{\forall x\phi(x)}}{D^i_{\phi(x/u)}}\ u \text{ fresh} \qquad \frac{A^i_{\exists x\phi(x)}}{A^i_{\phi(x/u)}}\ u \text{ fresh} \quad \frac{D^i_{\exists x\phi(x)}}{D^i_{\phi(x/c)}}$$

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Tableau rules

Closure conditions and $\vdash^{ij}$

Assertoric semantics

Why the strict-tolerant distinction is primitive

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Here are the tableau rules of our calculus, where $i \in \{s, t\}$:

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$$\frac{A^i_{T(\overline{\sigma})}}{A^i_{\sigma}} \qquad \frac{D^i_{T(\overline{\sigma})}}{D^i_{\sigma}}$$

$$\frac{A^i_{\forall x\phi(x)}}{A^i_{\phi(x/c)}} \quad \frac{D^i_{\forall x\phi(x)}}{D^i_{\phi(x/u)}}\ u \text{ fresh} \qquad \frac{A^i_{\exists x\phi(x)}}{A^i_{\phi(x/u)}}\ u \text{ fresh} \quad \frac{D^i_{\exists x\phi(x)}}{D^i_{\phi(x/c)}}$$

The $A^t_{\neg}$ rule is valid: $V(\neg\alpha) \in \{\frac{1}{2}, 1\} \Rightarrow V(\alpha) \in \{0, \frac{1}{2}\}$

Tableau rules

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Tableau rules

Closure conditions and $\vdash^{ij}$

Assertoric semantics

Why the strict-tolerant distinction is primitive

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$$\frac{A^i_{T(\overline{\sigma})}}{A^i_\sigma} \qquad \frac{D^i_{T(\overline{\sigma})}}{D^i_\sigma}$$

$$\frac{A^i_{\forall x\phi(x)}}{A^i_{\phi(x/c)}} \quad \frac{D^i_{\forall x\phi(x)}}{D^i_{\phi(x/u)}}\ u \text{ fresh} \qquad \frac{A^i_{\exists x\phi(x)}}{A^i_{\phi(x/u)}}\ u \text{ fresh} \quad \frac{D^i_{\exists x\phi(x)}}{D^i_{\phi(x/c)}}$$

The $A^t_\neg$ rule is valid: $V(\neg\alpha) \in \{\frac{1}{2}, 1\} \Rightarrow V(\alpha) \in \{0, \frac{1}{2}\}$

The $D^s_\vee$ rule is valid: $V(\alpha \vee \beta) = 0 \Rightarrow V(\alpha) = 0 \;\&\; V(\beta) = 0$, etc.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

Tableau rules

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The $D^s_\vee$ rule is valid: $V(\alpha \vee \beta) = 0 \Rightarrow V(\alpha) = 0 \;\&\; V(\beta) = 0$, etc.

Observe: no strict-to-tolerant rules.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

- A set of assertoric sentences S is closed iff:

- For some sentence σ of L_T: $\{A^s_\sigma, D^s_\sigma\} \subseteq S$
- For some truth-free sentence σ of L: $\{A^t_\sigma, D^t_\sigma\} \subseteq S$
- For some sentence σ of L_T: $\{A^s_\sigma, D^t_\sigma\} \subseteq S$
- For some sentence σ of L_T: $\{A^t_\sigma, D^s_\sigma\} \subseteq S$

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

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  - For some sentence σ of L_T: $\{A^s_\sigma, D^t_\sigma\} \subseteq S$
  - For some sentence σ of L_T: $\{A^t_\sigma, D^s_\sigma\} \subseteq S$

- When S is closed it is not fixed point satisfiable.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- For some sentence σ of L_T: $\{A^t_\sigma, D^s_\sigma\} \subseteq S$

- When S is closed it is not fixed point satisfiable.
- We define 4 syntactic SK consequence relations $\vdash^{ij}$, e.g.:

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

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  - For some truth-free sentence σ of L: $\{A^t_\sigma, D^t_\sigma\} \subseteq S$
  - For some sentence σ of L_T: $\{A^s_\sigma, D^t_\sigma\} \subseteq S$
  - For some sentence σ of L_T: $\{A^t_\sigma, D^s_\sigma\} \subseteq S$
- When S is closed it is not fixed point satisfiable.
- We define 4 syntactic SK consequence relations $\vdash^{ij}$, e.g.:
  - $\Gamma \vdash^{ss} \Delta \Leftrightarrow A^s(\Gamma) \cup D^t(\Delta)$ has a closed tableau.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

- A set of assertoric sentences S is closed iff:
  - For some sentence σ of L_T: $\{A^s_\sigma, D^s_\sigma\} \subseteq S$
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Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Tableau rules
Closure conditions and $\vdash^{ij}$
Assertoric semantics
Why the strict-tolerant distinction is primitive

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- **Theorem**: $\vdash^{ij}$ is sound and complete w.r.t. $\vDash^{ij}$.

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$

Why the strict-tolerant distinction is primitive

Assertoric semantics

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$

Why the strict-tolerant distinction is primitive

- The strict-tolerant calculus characterizes SK *consequence* in strict-tolerant terms.

The semantic counterpart of the strict-tolerant calculus

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

- The strict-tolerant calculus characterizes SK *consequence* in strict-tolerant terms.
- But, which sentences are *actually* strictly/tolerantly assertible/deniable? What about 'snow is white'? What about the Truthteller?

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

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- Given a fixed ground model M: what is the strict/tolerant assertoric status of L_T sentences?

The semantic counterpart of the strict-tolerant calculus

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$

Why the strict-tolerant distinction is primitive

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Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

The semantic counterpart of the strict-tolerant calculus

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- Quantifiers now range over the domain of M.

The semantic counterpart of the strict-tolerant calculus

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$

Why the strict-tolerant distinction is primitive

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- Quantifiers now range over the domain of M.

- Closure conditions of the strict-tolerant calculus are *augmented*:

Not allowed to assert (strictly or tolerantly) σ of L if $\mathcal{C}_M(\sigma) = 0$.

Not allowed to deny (strictly or tolerantly) σ of L if $\mathcal{C}_M(\sigma) = 1$.

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics
- The semantic counterpart of the strict-tolerant calculus
- Example: strict assertoric trees
- Example: tolerant assertoric trees
- Inducing familiar valuations
- Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$

Why the strict-tolerant distinction is primitive

- Let $\sigma := \neg T(\lambda) \wedge W(s)$. Is σ strictly assertible /deniable?

Example: strict assertoric trees

- Let $\sigma := \neg T(\lambda) \wedge W(s)$. Is σ strictly assertible /deniable?
- To answer, compute **strict assertoric trees**: $\mathfrak{T}^{\sigma}_{A^s}$ and $\mathfrak{T}^{\sigma}_{D^s}$:

$$
\begin{array}{c}
A^s_{\neg T(\lambda)\wedge W(s)} \\
| \\
A^s_{\neg T(\lambda)} \\
| \\
A^s_{W(s)} \\
| \\
D^s_{T(\lambda)} \\
| \\
D^s_{\neg T(\lambda)} \\
| \\
A^s_{T(\lambda)}
\end{array}
\qquad\qquad
\begin{array}{ccc}
 & D^s_{\neg T(\lambda)\wedge W(s)} & \\
\swarrow & & \searrow \\
D^s_{\neg T(\lambda)} & & D^s_{W(s)} \\
| & & \\
A^s_{T(\lambda)} & & \\
| & & \\
A^s_{\neg T(\lambda)} & & \\
| & & \\
D^s_{T(\lambda)} & &
\end{array}
$$

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

Example: strict assertoric trees

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| & & \\
A^s_{T(\lambda)} & & \\
| & & \\
A^s_{\neg T(\lambda)} & & \\
| & & \\
D^s_{T(\lambda)} & &
\end{array}
$$

- Both $\mathfrak{T}^\sigma_{A^s}$ and $\mathfrak{T}^\sigma_{D^s}$ are closed$_M$.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

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| \\
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| \\
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A^s_{T(\lambda)}
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 \swarrow & & \searrow \\
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| & & \\
A^s_{T(\lambda)} & & \\
| & & \\
A^s_{\neg T(\lambda)} & & \\
| & & \\
D^s_{T(\lambda)} & &
\end{array}
$$

- Both $\mathfrak{T}^{\sigma}_{A^s}$ and $\mathfrak{T}^{\sigma}_{D^s}$ are closed$_M$.

- $\mathcal{V}^s_M(\sigma) = (0,0)$: σ neither strictly assertible nor deniable.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

- Let $\sigma := \neg T(\lambda) \wedge W(s)$. Is σ tolerantly assertible /deniable?

Example: tolerant assertoric trees

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

- The semantic counterpart of the strict-tolerant calculus
- Example: strict assertoric trees
- Example: tolerant assertoric trees
- Inducing familiar valuations
- Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$

Why the strict-tolerant distinction is primitive

- Let $\sigma := \neg T(\lambda) \wedge W(s)$. Is σ tolerantly assertible /deniable?
- To answer, compute **tolerant assertoric trees**: $\mathfrak{T}^\sigma_{A^t}$ and $\mathfrak{T}^\sigma_{D^t}$:

$\mathfrak{T}^\sigma_{A^t}$:

- $A^t_{\neg T(\lambda)\wedge W(s)}$
  - $A^t_{\neg T(\lambda)}$
    - $A^t_{W(s)}$
      - $D^t_{T(\lambda)}$
        - $D^t_{\neg T(\lambda)}$
          - $A^t_{T(\lambda)}$

$\mathfrak{T}^\sigma_{D^t}$:

- $D^t_{\neg T(\lambda)\wedge W(s)}$
  - $D^t_{\neg T(\lambda)}$
    - $A^t_{T(\lambda)}$
      - $A^t_{\neg T(\lambda)}$
        - $D^t_{T(\lambda)}$
  - $D^t_{W(s)}$

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

- Let $\sigma := \neg T(\lambda) \wedge W(s)$. Is σ tolerantly assertible /deniable?
- To answer, compute **tolerant assertoric trees**: $\mathfrak{T}^{\sigma}_{A^t}$ and $\mathfrak{T}^{\sigma}_{D^t}$:

$$\begin{array}{c} A^t_{\neg T(\lambda)\wedge W(s)} \\ | \\ A^t_{\neg T(\lambda)} \\ | \\ A^t_{W(s)} \\ | \\ D^t_{T(\lambda)} \\ | \\ D^t_{\neg T(\lambda)} \\ | \\ A^t_{T(\lambda)} \end{array} \qquad \begin{array}{ccc} & D^t_{\neg T(\lambda)\wedge W(s)} & \\ \swarrow & & \searrow \\ D^t_{\neg T(\lambda)} & & D^t_{W(s)} \\ | & & \\ A^t_{T(\lambda)} & & \\ | & & \\ A^t_{\neg T(\lambda)} & & \\ | & & \\ D^t_{T(\lambda)} & & \end{array}$$

- Both $\mathfrak{T}^{\sigma}_{A^t}$ and $\mathfrak{T}^{\sigma}_{D^t}$ are open$_M$.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

Example: tolerant assertoric trees

- Let $\sigma := \neg T(\lambda) \wedge W(s)$. Is σ tolerantly assertible /deniable?
- To answer, compute **tolerant assertoric trees**: $\mathfrak{T}^\sigma_{A^t}$ and $\mathfrak{T}^\sigma_{D^t}$:

$\mathfrak{T}^\sigma_{A^t}$:

- $A^t_{\neg T(\lambda)\wedge W(s)}$
  - $A^t_{\neg T(\lambda)}$
    - $A^t_{W(s)}$
      - $D^t_{T(\lambda)}$
        - $D^t_{\neg T(\lambda)}$
          - $A^t_{T(\lambda)}$

$\mathfrak{T}^\sigma_{D^t}$:

- $D^t_{\neg T(\lambda)\wedge W(s)}$
  - $D^t_{\neg T(\lambda)}$
    - $A^t_{T(\lambda)}$
      - $A^t_{\neg T(\lambda)}$
        - $D^t_{T(\lambda)}$
  - $D^t_{W(s)}$

- Both $\mathfrak{T}^\sigma_{A^t}$ and $\mathfrak{T}^\sigma_{D^t}$ are open$_M$.
- $\mathcal{V}^t_M(\sigma) = (1,1)$: σ both tolerantly assertible and deniable.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

- With M a ground model, $\mathcal{V}^s_M$ and V^t_M are induced as follows. With $i, j \in \{s, t\}$:

$$\mathcal{V}^i_M(\sigma) = \begin{cases} (1,0), & \mathfrak{T}^\sigma_{A^i} \text{ is open}_M \;\&\; \mathfrak{T}^\sigma_{D^i} \text{ is closed}_M \\ (1,1), & \mathfrak{T}^\sigma_{A^i} \text{ is open}_M \;\&\; \mathfrak{T}^\sigma_{D^i} \text{ is open}_M \\ (0,0), & \mathfrak{T}^\sigma_{A^i} \text{ is closed}_M \;\&\; \mathfrak{T}^\sigma_{D^i} \text{ is closed}_M \\ (0,1), & \mathfrak{T}^\sigma_{A^i} \text{ is closed}_M \;\&\; \mathfrak{T}^\sigma_{D^i} \text{ is open}_M \end{cases}$$

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

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- **Theorem** $\mathcal{V}^s_M$ is equivalent to Kripke's $\mathcal{K}^{\mathbf{4}}_M$, where:

$$\mathcal{K}^{\mathbf{4}}_M(\sigma) = (1,0) \Leftrightarrow \exists V_M : V_M(\sigma) = 1 \text{ and } \nexists\, V_M : V_M(\sigma) = 0$$
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Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

Inducing familiar valuations

- With M a ground model, $\mathcal{V}^s_M$ and V^t_M are induced as follows. With $i, j \in \{s, t\}$:

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- **Theorem** $\mathcal{V}^t_M : Sen(L_T) \to \{(1,0), (1,1), (0,1)\}$ is equivalent to the minimal fixed point over M.

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$

Why the strict-tolerant distinction is primitive

- With $T(\tau)$ a Truthteller: $\mathcal{V}^s_M(T(\tau)) = (1,1)$.

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$

Why the strict-tolerant distinction is primitive

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- So the Truthteller is both strictly assertible and deniable.

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$

Why the strict-tolerant distinction is primitive

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- But $A^s_{T(\tau)}$ and $D^s_{T(\tau)}$ give closure?

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$

Why the strict-tolerant distinction is primitive

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Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

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Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

The semantic counterpart of the strict-tolerant calculus

Example: strict assertoric trees

Example: tolerant assertoric trees

Inducing familiar valuations

Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$

Why the strict-tolerant distinction is primitive

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Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$
Why the strict-tolerant distinction is primitive

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- One may not become committed to both a strict assertion and strict denial of $T(\tau)$, as one does e.g. by asserting $T(\tau) \wedge \neg T(\tau)$.
- $\mathcal{V}^s_M(T(\tau) \wedge \neg T(\tau)) = (0,1)$

■ $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$ model *initial* assertoric possibilities. Performing strict/tolerant assertoric actions rules out other such actions.

Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
The semantic counterpart of the strict-tolerant calculus
Example: strict assertoric trees
Example: tolerant assertoric trees
Inducing familiar valuations
Some remarks on the interpretation of $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$
Why the strict-tolerant distinction is primitive

- With $T(\tau)$ a Truthteller: $\mathcal{V}_M^s(T(\tau)) = (1, 1)$.

- So the Truthteller is both strictly assertible and deniable.

- But $A^s_{T(\tau)}$ and $D^s_{T(\tau)}$ give closure?

- One may strictly assert $T(\tau)$ and one may strictly deny $T(\tau)$, but one may not do so ”at the same time”.

- One may not become committed to both a strict assertion and strict denial of $T(\tau)$, as one does e.g. by asserting $T(\tau) \wedge \neg T(\tau)$.

- $\mathcal{V}_M^s(T(\tau) \wedge \neg T(\tau)) = (0, 1)$

- $\mathcal{V}_M^s$ and $\mathcal{V}_M^t$ model *initial* assertoric possibilities. Performing strict/tolerant assertoric actions rules out other such actions.
- The transmission of assertoric possibilities due to (strict and tolerant) assertions and denials is captured by the strict-tolerant calculus.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

Why the strict-tolerant distinction is primitive

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

- We interpret A^s, D^s, A^t and D^t as *force indicators*.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

- We interpret A^s, D^s, A^t and D^t as *force indicators*.
- Why does Ripley think that STCT has *bi*laterlist and not, say *four*lateralist commitments?

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

- We interpret A^s, D^s, A^t and D^t as *force indicators*.
- Why does Ripley think that STCT has *bi*laterlist and not, say *four*lateralist commitments?

> [The strict-tolerant distinction] it is not a primitive distinction; we can understand tolerant assertion and denial in terms of their strict cousins, as I've presented them here, or we can equally well understand strict in terms of tolerant. So long as we have a grip on one, there is no difficulty in coming to understand the other. Ripley, PaFC

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

- We interpret A^s, D^s, A^t and D^t as *force indicators*.
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- In a sense, this remark is to the point. But not in the required sense.

Why *bi*lateralism?

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

- We interpret A^s, D^s, A^t and D^t as *force indicators*.
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- In a sense, this remark is to the point. But not in the required sense.
- The remark is to the point relative to a particular fixed point V_M:

σ is strongly$_{V_M}$ assertible $\Leftrightarrow$ σ is not tolerantly$_{V_M}$ deniable
σ is strongly$_{V_M}$ deniable $\Leftrightarrow$ σ is not tolerantly$_{V_M}$ assertible

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

- The strict and tolerant can be understood in terms of one another if, given M, there would be a *privileged* V^*_M which would inform us about the assertoric status of the L_T sentences.

Supervenience of Semantics and the Fixed Point Conception

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

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- **Supervenience of semantics**: Once all the empirical facts have been settled, so are all the semantic facts. In terms of our formal theory, the intuition becomes: for any given ground model, there is *exactly one* correct interpretation of the truth predicate. M. Kremer 1988

Supervenience of Semantics and the Fixed Point Conception

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

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- **Fixed Point Conception of Truth**: This criterion takes the notion of a fixed point to give the whole meaning of true. Or, in Kripke's words, the intuitive concept of truth is expressed by the formula: 'we are entitled to assert (or deny) of a sentence that it is true precisely under the circumstances when we can assert (or deny) the sentence itself'. M. Kremer 1988

Supervenience of Semantics and the Fixed Point Conception

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Why *bi*lateralism?

Supervenience of Semantics and the Fixed Point Conception

But not so fast. . . Argument 1

But not so fast. . . Argument 2

But not so fast. . . Argument 3

Concluding remarks

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Supervenience of Semantics and the Fixed Point Conception

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Why *bi*lateralism?

Supervenience of Semantics and the Fixed Point Conception

But not so fast... Argument 1

But not so fast... Argument 2

But not so fast... Argument 3

Concluding remarks

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- Due to its inferentialist commitments, STCT is committed to **FPCT**.

- Moreover, as $\mathcal{V}^s_M$ and $\mathcal{V}^t_M$ do not determine each other, we can't understand the strict in terms of tolerant (nor vice versa).

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

- Thus, we need 4 distinct primitive speech acts?

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Why *bi*lateralism?

Supervenience of Semantics and the Fixed Point Conception

But not so fast. . . Argument 1

But not so fast. . . Argument 2

But not so fast. . . Argument 3

Concluding remarks

- Thus, we need 4 distinct primitive speech acts?
- Not so fast. For look at assertoric semantics:

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

- Thus, we need 4 distinct primitive speech acts?
- Not so fast. For look at assertoric semantics:

- Modulo an insignificant difference in sign: $\mathfrak{T}^{\sigma}_{A^s} = \mathfrak{T}^{\sigma}_{A^t}$ and $\mathfrak{T}^{\sigma}_{D^s} = \mathfrak{T}^{\sigma}_{D^t}$

But not so fast... Argument 1

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

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  - Thus, there are only 2 speech acts (assertion and denial) but two distinct assertoric norms (closure conditions): a strict and a tolerant one.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

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- However, assertoric semantics does not take into account the relations between strict and tolerant.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

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  - Thus, there are only 2 speech acts (assertion and denial) but two distinct assertoric norms (closure conditions): a strict and a tolerant one.
- However, assertoric semantics does not take into account the relations between strict and tolerant.
- The strict-tolerant calculus does so in its closure conditions:

  A^s_σ, D^t_σ or A^t_σ, D^s_σ occur on a tableau path.

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Why *bi*lateralism?

Supervenience of the Semantics and the Fixed Point Conception

But not so fast. . . Argument 1

But not so fast. . . Argument 2

But not so fast. . . Argument 3

Concluding remarks

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- Not so fast. For look at assertoric semantics:
  - Modulo an insignificant difference in sign: $\mathfrak{T}^{\sigma}_{A^s} = \mathfrak{T}^{\sigma}_{A^t}$ and $\mathfrak{T}^{\sigma}_{D^s} = \mathfrak{T}^{\sigma}_{D^t}$
  - Thus, there are only 2 speech acts (assertion and denial) but two distinct assertoric norms (closure conditions): a strict and a tolerant one.
- However, assertoric semantics does not take into account the relations between strict and tolerant.
- The strict-tolerant calculus does so in its closure conditions:

  A^s_σ, D^t_σ or A^t_σ, D^s_σ occur on a tableau path.
- Hence, in order to understand the relations between strict and tolerant actions, it seems that we must understand the signs A^s, D^s, A^t, D^t as primitive force indicators.

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Why *bi*lateralism?

Supervenience of Semantics and the Fixed Point Conception

But not so fast. . . Argument 1

But not so fast. . . Argument 2

But not so fast. . . Argument 3

Concluding remarks

- Perhaps then, the strict and tolerant can't be understood in terms of one another.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

- Perhaps then, the strict and tolerant can't be understood in terms of one another.

- But who cares? What is at the heart of STCT is a syntactic (bilaterlistic) characterization of $\models^{st}$. The strict and tolerant are at least *on a par* as we can do so by putting constraints either on:

But not so fast... Argument 2

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

- Perhaps then, the strict and tolerant can't be understood in terms of one another.
- But who cares? What is at the heart of STCT is a syntactic (bilaterlistic) characterization of $\vDash^{st}$. The strict and tolerant are at least *on a par* as we can do so by putting constraints either on:

  -strict assertions and strict denials
  -tolerant assertions and tolerant denials.
- *According to the strict-tolerant calculus*, this argument is wrong.

But not so fast... Argument 2

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

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- *According to the strict-tolerant calculus*, this argument is wrong.
- $\vDash^{st}$ can be characterized by putting constraints on strict assertions and denials (as in $\vdash^{st}$) but not in terms their tolerant cousins.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

But not so fast... Argument 2

- Perhaps then, the strict and tolerant can't be understood in terms of one another.

- But who cares? What is at the heart of STCT is a syntactic (bilaterlistic) characterization of $\models^{st}$. The strict and tolerant are at least *on a par* as we can do so by putting constraints either on:

  -strict assertions and strict denials
  -tolerant assertions and tolerant denials.

- *According to the strict-tolerant calculus*, this argument is wrong.

- $\models^{st}$ can be characterized by putting constraints on strict assertions and denials (as in $\vdash^{st}$) but not in terms their tolerant cousins.

- According to the strict-tolerant calculus, strict assertions and denials have a privileged status.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

- But we can easily restore the asymmetry between the strict and tolerant by characterizing $\vDash^{st}$ via the notion of *refusal*:

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

- But we can easily restore the asymmetry between the strict and tolerant by characterizing $\vDash^{st}$ via the notion of *refusal*:
- Refusal-to-tolerantly-deny all premisses and refusal-to-tolerantly-assert all consequences is out of bounds

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

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- Refusal-to-tolerantly-deny all premisses and refusal-to-tolerantly-assert all consequences is out of bounds
- Refusal-to-tolerantly-deny σ is tantamount to strictly asserting σ.

But not so fast. . . Argument 3

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Why *bi*lateralism?

Supervenience of Semantics and the Fixed Point Conception

But not so fast. . . Argument 1

But not so fast. . . Argument 2

But not so fast. . . Argument 3

Concluding remarks

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- But what kind of refusal is this?

But not so fast... Argument 3

Introduction

From classical to Strong Kleene consequence

The strict-tolerant calculus

Assertoric semantics

Why the strict-tolerant distinction is primitive

Why *bi*lateralism?

Supervenience of Semantics and the Fixed Point Conception

But not so fast... Argument 1

But not so fast... Argument 2

But not so fast... Argument 3

Concluding remarks

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- $\models^{st} \sigma, \neg\sigma$. Out of bounds to refuse to (tolerantly) assert σ and its negation. But what if σ is unknown?

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

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Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

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- $\models^{st} \sigma, \neg\sigma$. Out of bounds to refuse to (tolerantly) assert σ and its negation. But what if σ is unknown?
- Go multi-agent? If agent A refuses B's tolerant assertion of σ, A is committed to a strict denial of σ.
- Then again, even a detailed notion of refusal (and acceptance) only shows that the strict-tolerant symmetry can be ”restored” via auxiliary notions.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

- The idea of advocating a non-transitive consequence relation for L_T (and vagueness) is, I take it, a very interesting and promising direction.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast. . . Argument 1
But not so fast. . . Argument 2
But not so fast. . . Argument 3
Concluding remarks

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- The idea of advocating a non-transitive consequence relation for L_T (and vagueness) is, I take it, a very interesting and promising direction.
- Same for STCT, who seeks to defend this relation philosophically by deriving it from an independent account of meaning (bilateralism).

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

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- The idea of advocating a non-transitive consequence relation for L_T (and vagueness) is, I take it, a very interesting and promising direction.
- Same for STCT, who seeks to defend this relation philosophically by deriving it from an independent account of meaning (bilateralism).
- The strict-tolerant calculus and assertoric semantics shed light on STCT and suggest that the strict-tolerant distinction needs further attention.

Introduction
From classical to Strong Kleene consequence
The strict-tolerant calculus
Assertoric semantics
Why the strict-tolerant distinction is primitive
Why *bi*lateralism?
Supervenience of Semantics and the Fixed Point Conception
But not so fast... Argument 1
But not so fast... Argument 2
But not so fast... Argument 3
Concluding remarks

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- Same for STCT, who seeks to defend this relation philosophically by deriving it from an independent account of meaning (bilateralism).
- The strict-tolerant calculus and assertoric semantics shed light on STCT and suggest that the strict-tolerant distinction needs further attention.
- Thanks for your attention.