An introduction to Borel reducibility

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Abstract. In a seminal paper in 1989, Friedman and Stanley introduced Borel reducibility for countable structures. The idea is to measure how hard it is to determine whether two structures in a class are isomorphic. We say that a class of structures $\mathcal{C}$ is Borel reducible to a class of structures $\mathcal{D}$ if given any structure $\mathcal{A}$ in $\mathcal{C}$ we can build, in a Borel (or in practice, usually computable) way, a structure $F(\mathcal{A})$ in $\mathcal{D}$ such that two structures $\mathcal{A}$ and $\mathcal{B}$ are isomorphic if and only if $F(\mathcal{A})$ and $F(\mathcal{B})$ are isomorphic. The idea is that if you knew how to decide whether two structures in $\mathcal{D}$ were isomorphic, then you could decide whether two structures in $\mathcal{C}$ were isomorphic. I will introduce this reducibility and we will see some of the standard examples. Some of the topics we will discuss include comparisons to graph isomorphism in complexity theory and the stronger notion of universality in computability theory. In the second talk of the tutorial, I will discuss the recent proofs that torsion-free abelian groups are Borel-complete.