

Implicative algebras: a survey

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Abstract. Implicative algebras are a generalization of complete Heyting algebras intended to factorize the model-theoretic constructions underlying forcing and realizability, both in intuitionistic and classical logic. In this talk, I will first recall the definition of this structure, that relies on the identification of the notion of truth value with the notion of realizer. Then, I will show that with the suitable notion of morphism, implicative algebras form a locally small category that is equivalent to (and thus can be used to represent) a particular category of functors, namely: the category of Set-based triposes (in the sense of Hyland, Johnstone and Pitts) equipped with finite meet-preserving natural transformations. Regarding set theory, I will show that implicative algebras can also be used to construct models of (intuitionistic and classical) set theory, that encompass Boolean-valued models of ZF as well as intuitionistic and classical realizability models of (I)ZF. I will conclude by presenting some perspectives of using implicative algebras to interpret first-order theories.