Topology of Science: Empirical, Metaphysical, Erotetic (book draft)

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Philosophy of Science

Relevance
- What matters
- Type of inference problem
- Lower bound on content
Philosophy of Science

Empirical information
• Verification
• Refutation
• Underdetermination
• Learnability
• Simplicity and Ockham’s razor

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• What matters
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Reality
• Natural science vs. Data-science
• Miracles and Luck
• Negligibility

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Three Topological Bases

- Metaphysical $\mathcal{M}$
- Empirical $\mathcal{E}$
- Erotetic $\mathcal{Q}$
Three Topologies

Metaphysical

$\mathcal{M}^*$

Empirical

$\mathcal{E}^*$

Erotetic

$\mathcal{Q}^*$
Three Topologies

Topology of real similarity

Topology of empirical similarity

Topology of relevant similarity
Context of Inquiry

$$\mathfrak{C} = (W, \mathcal{M}, \mathcal{E}, Q)$$
1. EROTETIC BASIS
Erotetic Basis

- Elements of $Q$ are answers to a question.
- Answers should be concluded eventually, if true.

$Q$ is a countable topological basis.

1. At worst, vacuous information $W$ is requested.
2. Requests accumulate.
3. Requests are expressible.
Erotetic Basis

Allows for overlapping answers.

1. One-sided questions:
   verification = \{A, W\};
   refutation = \{\neg A, W\},
   decision = \{A, \neg A\}.

2. Replace hopeless catch-all hypothesis with $W$.

3. Quantitative (estimation) questions have open intervals as answers.

# Erotetic Operators

<table>
<thead>
<tr>
<th>Inquiry</th>
<th>Erotetic Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>“You should conclude $A$”</td>
<td>$\text{int } A$</td>
</tr>
<tr>
<td>“You should deny $A$”</td>
<td>$\text{ext } A$</td>
</tr>
<tr>
<td>“You needn’t deny $A$”</td>
<td>$\text{cl } A$</td>
</tr>
<tr>
<td>“You needn’t decide $A$”</td>
<td>$\text{bdry } A$</td>
</tr>
<tr>
<td>“You needn’t conclude $A$, even though it’s true”</td>
<td>$\text{frnt } \neg A$</td>
</tr>
</tbody>
</table>
## Erotetic Properties

<table>
<thead>
<tr>
<th>Inquiry</th>
<th>Erotetic Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>“$A$ is positively relevant”</td>
<td>$A$ is open</td>
</tr>
<tr>
<td>“$A$ is negatively relevant”</td>
<td>$A$ is closed</td>
</tr>
<tr>
<td>“$A$ is relevant”</td>
<td>$A$ is clopen</td>
</tr>
</tbody>
</table>
2. EMPIRICAL BASIS
Empirical Basis

• Elements of $\mathcal{E}$ are empirical information states.

$\mathcal{E}$ is a countable topological basis.

1. At worst, vacuous information $\mathcal{W}$ is available.
2. Available information accumulates.
3. Scientific information is recordable.

$\mathcal{E}_W = \text{the set of all empirical information states true/possible in } \mathcal{W}$. 
Serendipity

- $E \in \mathcal{E}_w$ says you might obtain $E$ in $w$ by luck.
Achievable Information

- $E \in \mathcal{E}_w$ says diligence \textit{will} yield at least $E$ in $w$.
- Familiar \textit{normative} requirement on experimental results.
## Empirical Modalities

<table>
<thead>
<tr>
<th>Inquiry</th>
<th>Empirical Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>“A will be verified”</td>
<td>int $A$</td>
</tr>
<tr>
<td>“A will be refuted”</td>
<td>ext $A$</td>
</tr>
<tr>
<td>“A will be decided”</td>
<td>$\neg$bdry $A$</td>
</tr>
<tr>
<td>“A will never be verified”</td>
<td>cl$\neg A$</td>
</tr>
<tr>
<td>“A will never be refuted”</td>
<td>cl $A$</td>
</tr>
<tr>
<td>“A will never be decided”</td>
<td>bdry $A$</td>
</tr>
<tr>
<td>“A is false but will never be refuted” = Popper’s problem</td>
<td>frnt $A$ ($=\text{cl } A \setminus A$)</td>
</tr>
<tr>
<td>“A is true but will never be verified” = Hume’s problem</td>
<td>frnt $\neg A$</td>
</tr>
</tbody>
</table>
Empirical Properties

<table>
<thead>
<tr>
<th>Inquiry</th>
<th>Erotetic Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>“A is verifiable”</td>
<td>A is open</td>
</tr>
<tr>
<td>“A is refutable”</td>
<td>A is closed</td>
</tr>
<tr>
<td>“A is decidable”</td>
<td>A is clopen</td>
</tr>
<tr>
<td>“A is verifutable”</td>
<td>A is locally closed</td>
</tr>
</tbody>
</table>
## Ero-Empirical Modalities

<table>
<thead>
<tr>
<th>Inquiry</th>
<th>Ero-Empirical Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>“A will be <em>irrelevantly</em> verified”</td>
<td>$\text{int } A \setminus \text{int } A$</td>
</tr>
<tr>
<td>“The problem of induction arises <em>relevantly</em> for $A$”</td>
<td>$\text{frnt} \neg A \cap \text{int } A$</td>
</tr>
<tr>
<td>Etc.</td>
<td></td>
</tr>
</tbody>
</table>
**Learnability**

<table>
<thead>
<tr>
<th>Inquiry</th>
<th>Empirical Topology</th>
</tr>
</thead>
<tbody>
<tr>
<td>“$Q$ is answerable infallibly”</td>
<td>Each answer to $Q$ is $\mathcal{E}$-open</td>
</tr>
<tr>
<td>“$Q$ is answerable in the limit with elimination of false reasons”</td>
<td>Each answer to $Q$ is $\mathcal{E}$-sigma-constructible (= countable union of differences of opens)</td>
</tr>
<tr>
<td>“$Q$ is answerable in the limit with elimination of false answers”</td>
<td>Each answer to $Q$ is $\mathcal{E}$-sigma-constructible and co-sigma-constructible.</td>
</tr>
</tbody>
</table>
Popper’s Analysis of Simplicity

\[ A \leq B \]

iff every information state that falsifies \( B \) falsifies \( A \).

iff \( A \subseteq \text{cl} \ B \).

Two flaws.

1. \( W \) is strictly more complex than every other proposition, so mere suspension of judgment violates Ockham’s razor!

2. Maybe \( A \) is simpler than \( B \) somewhere but not everywhere.

LOL, It’s topological!
Empirical Simplicity

Improvement:

• \( A \triangleleft B = \text{“} A \text{ is strictly simpler than } B \text{”} \)
  \[ = A \cap \text{frnt } B. \]

• \( A \) is Ockham given \( E \) iff
  no \( B \) is possibly simpler than \( A \) given \( E \).

**Prop.** The following are equivalent.

1. \( A \) is Ockham given \( E \).
2. \( A \) is closed (refutable) given \( E \).
Ockham Necessity Theorem

**Prop.** Suppose that method $M$:

- answers $Q$,
- eliminates false reasons,
- never drops a true reason.

Then $M$ concludes an **Ockham reason** for each answer.
3. METAPHYSICAL BASIS
Scientific Realism

- **Scientific realists** think science can penetrate beneath the appearances.

- To address realism, one must represent hidden reality.
Metaphysical Basis

• Worlds are more or less similar.
• $\rho$ is the (dis-)similarity metric.

• $\mathcal{M}$ is the set of all open metric balls.
Nice, but Hopeless

Which world is more similar to $c$?
Metaphysical Topology

- Hopeless comparisons are sidestepped by the induced topology $\mathcal{M}^*$.  
  - Across models, discrete difference. 
  - Within models, standard metric topology.
  - That determines the metaphysical topology uniquely.
# Metaphysical Modalities

<table>
<thead>
<tr>
<th>Inquiry</th>
<th>Topology</th>
</tr>
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<tbody>
<tr>
<td>“A is securely true.”</td>
<td><code>int A</code></td>
</tr>
<tr>
<td>“A is securely false.”</td>
<td><code>ext A</code></td>
</tr>
<tr>
<td>“A has a secure truth value.”</td>
<td><code>¬bdry A</code></td>
</tr>
<tr>
<td>“A is or is arbitrarily close to being false.”</td>
<td><code>cl ¬A</code></td>
</tr>
<tr>
<td>“A is or is arbitrarily close to being true.”</td>
<td><code>cl A</code></td>
</tr>
<tr>
<td>“A is brittle.”</td>
<td><code>bdry A</code></td>
</tr>
<tr>
<td>“A is barely false.”</td>
<td><code>frnt A</code></td>
</tr>
<tr>
<td>“A is barely true.”</td>
<td><code>frnt ¬A</code></td>
</tr>
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## Metaphysical Properties

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<tr>
<td>“$A$ is natural” = “$A$ cannot be barely true”</td>
<td>$A$ is open</td>
</tr>
</tbody>
</table>

**Natural:**
- Open interval estimates (properly open).
- Models (clopen).
- Paradigms = countable disjunctions of models (clopen).

**Unnatural:**
- Arbitrary thresholds.
- Arbitrary quantitative models with no interpretation.
- Arbitrary parameter settings.
Clockwise

**Finite Precision**
Arbitrary similarity is not resolvable. Axiom.

**Relevant Reality**
Avoid irrelevant details
Pragmatic semantic advice.

**Deductive Question**
Answers are verifiable
Special case or inductive skepticism
Counter-Clockwise

Operationism
Every real difference is empirically resolvable
Contentious thesis

Natural Question
Irrelevance of slippery slope distinctions
Axiom of natural science

Data Retention
Information states are relevant
Axiom of natural science

 karşıs

Axiom of natural science

Irrelevance of slippery slope distinctions
Axiom of natural science
### Transitive Implications

<table>
<thead>
<tr>
<th>Finite Precision</th>
<th>∧</th>
<th>Relevant Reality</th>
<th>⟹</th>
<th>Data Retention</th>
</tr>
</thead>
<tbody>
<tr>
<td>Finite Precision</td>
<td>∧</td>
<td>Deductive Question</td>
<td>⟹</td>
<td>Natural Question</td>
</tr>
<tr>
<td>Relevant Reality</td>
<td>∧</td>
<td>Deductive Question</td>
<td>⟹</td>
<td>Operationism</td>
</tr>
<tr>
<td>Operationism</td>
<td>∧</td>
<td>Natural Question</td>
<td>⟹</td>
<td>Deductive Question</td>
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<td>Operationism</td>
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<td>Data Retention</td>
<td>⟹</td>
<td>Relevant Reality</td>
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<tr>
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<td>∧</td>
<td>Natural Question</td>
<td>⟹</td>
<td>Finite Precision</td>
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</table>
All Contexts

Finite Precision

Data Retention

\[ \mathcal{M}^* \subseteq \mathcal{E}^* \cup \mathcal{Q}^* \]
Realist Contexts

Finite Precision $\cup$ $\mathcal{E}^*$ $\cup$ Natural Question $\subseteq$ Data Retention

$\mathcal{M}^*$
Deductive Contexts

Finite Precision $\cup$ $\mathcal{E}^*$ $\cup$ Natural Question $\mathcal{Q}^*$

Data Retention
Deductive Question

$\mathcal{M}^*$
Operationist Contexts

\[ \begin{align*}
Q^* & \Delta \equiv M^* \\
\Delta & \equiv E^* \\
\Delta & \equiv Q^* \\
\end{align*} \]

- Operationism
- Finite precision
- Data Retention
- Deductive Question
- Natural Question
- Relevance of Reality
MIRACLES AND FINE-TUNING
Fine-tuning

• Truth of $A$ teeters at the edge of a sea of falsehood.
• $\text{mir } A = A \cap \text{cl int } \neg A$. 
Miracles

• $A$ is miraculous iff $A \subseteq \text{mir } A$. 

\[ \text{int } \neg A \]
Famous Scientific Miracles

• The morning star is on the same orbit as the evening star.
• Mars’ epicycle is perfectly synchronized with the sun’s deferent.
• Reflecting telescopes produce exactly the same illusions as refracting telescopes.
• Light is distinct from EM radiation, even though they have exactly the same speed.
• It matters whether the coil or the magnet is moving, even though the current is exactly the same.
NEGLECT
Realists $\textbf{Neglect}$ Miracles

- Hidden realities go beyond all possible empirical information.
- Realism neglects $\textit{miraculous}$ possibilities of error.
- Anti-realism refuses to.
Metaphysical Negligibility

• \(A\) is nowhere dense iff \(\text{int cl } A = \emptyset\).

• The nowhere dense propositions are a non-trivial ideal.
  1. Closed under subset.
  2. Closed under finite union.
  3. Exclude \(W\).

• So nowhere density is a concept of negligibility.

• Unlike prior probability, it is a semantic/metaphysical concept of negligibility.
Realism Theorem

**Prop.** $A$ is nowhere dense iff $A$ is miraculous.

So the miraculous propositions are exactly the negligible ones!
Example: Theoretical Identification

\( X = \text{maximum elongation of Hesperus.} \)
\( Y = \text{maximum elongation of Phosphorus.} \)
\( A = \text{“the two planets are identical”.} \)
\( M = \text{“they are different, but } X = Y \text{ anyway”.} \)
Example: Theoretical Identification

$M$ is miraculous/negligible, so unnatural.

$A, \neg A$ are open/natural, so not miraculous/negligible.

Natural question: $\{A, \neg A\}$.
Realism Vindicated

- $M$ is empirically identical to $A$.
- But $M$ is negligible and $A$ is not.
- Neglecting $M$, Ockham’s razor mandates $A$. 

\[ X = Y \]