Bisimulations between Veltman models and generalized Veltman models

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- Hilbert-Bernays conditions and Löb’s theorem correspond to axioms and inference rules of $\mathbf{GL}$. 

$\rightarrow$ Some base theory $T$ extended by $A$ interprets $T$ extended by $B$.

Some known results on interpretability correspond to axioms of the basic interpretability logic $\mathbf{IL}$ (Visser 1988) and its extensions.

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$A \rightarrow (A \land \Box C) \rightarrow (B \land \Box C)$ (Montagna’s principle)
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Modal semantics

Veltman models:

- $W \neq \emptyset$
- $R \subseteq W \times W$ transitive and reverse well-founded
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- for each $w \in W$, $S_w \subseteq R[w] \times R[w]$

Satisfaction:

$w \models A \supset B$ if for all $u$ s.t. $wRu$ and $u \models A$ there is $v$ s.t. $uS_wv$ and $v \models B$
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- For each $w \in W$, $S_w \subseteq R[w] \times R[w]$
  - If $wRu$ then $uS_wu$
- Satisfaction: $w \models A \Rightarrow B$ if for all $u$ s.t. $wRu$ and $u \models A$ there is $v$ s.t. $uS_wv$ and $v \models B$
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Generalized semantics

Generalized Veltman models:

- $W \neq \emptyset$
- $R \subseteq W \times W$ transitive and reverse well-founded
- For each $w \in W$, $S_w \subseteq R[w] \times \mathcal{P}(R[w])$
  - if $wRu$ then $uS_w\{u\}$
  - if $uS_w V$ and $vS_w Z_v$ for all $v \in V$ then $uS_w(\bigcup Z_v)$
  - if $wRuRv$ then $uS_w\{v\}$

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Key properties:
▶ if $wZw'$, then $w$ and $w'$ are modally equivalent
▶ the converse does not hold generally, but it holds in case of image-finite Veltman models (an analogue of Hennessy-Milner theorem, de Jonge 2004)
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Now, as desired:

▶ bisimilarity implies modal equivalence
▶ Hennessy-Milner analogue holds
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Let \( W \) be a generalized Veltman model and \( W' \) a Veltman model. A bisimulation is \( Z \subseteq W \times W' \) s.t.

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- bisimilarity implies modal equivalence
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Example

Consider a generalized Veltman frame such that:

- $W = \{0, 1, 2, 3\}$, $R = \{(0, 1), (0, 2), (0, 3)\}$, $1S_0\{2, 3\}$
- $1 \models p$, $2 \models q$, $3 \models r$

Then $Z = \{(0, 0'), (1, 1'), (1, 1''), (2, 2'), (3, 3')\}$ is a bisimulation.

Hence, $0$ and $0'$ are modally equivalent (as are all pairs in $Z$).

With the more restrictive definition of bisimulation, we would not have a bisimulation in this example, thus we can use it as a counterexample for Hennessy-Milner analogue in that case.
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Now, consider a Veltman frame as follows:

- \( W = \{0', 1', 1'', 2', 3'\} \), \( R = \{(0', 1'), (0', 1''), (0', 2'), (0', 3')\} \), \( 1'S_0'2', 1''S_0'3' \)
- \( 1' \models p \), \( 1'' \models p \), \( 2' \models q \), \( 3' \models r \)

Then \( Z = \{(0, 0'), (1, 1'), (1, 1''), (2, 2'), (3, 3')\} \) is a bisimulation. Hence, \( 0 \) and \( 0' \) are modally equivalent (as are all pairs in \( Z \)). With the more restrictive definition of bisimulation, we would not have a bisimulation in this example, thus we can use it as a counterexample for Hennessy-Milner analogue in that case.
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Then $\mathcal{Z} = \{(0, 0'), (1, 1'), (1, 1''), (2, 2'), (3, 3')\}$ is a bisimulation. Hence, 0 and 0' are modally equivalent (as are all pairs in $\mathcal{Z}$).
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Obtaining a bisimilar model

It is straightforward to obtain a bisimilar generalized Veltman model from a given Veltman model: we use the same $W$ and $R$, and define $uS'_w V$ iff $uS_w v$ for some $v \in V$. 
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The previous example is very simple, but already illustrates that the opposite direction is much more involved. Exploring it is an ongoing work.