

On the axiomatizability of modal many-valued logics

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Modal logic is one of the most developed and studied non-classical logics, yielding a beautiful equilibrium between complexity and expressibility. The idea of enriching a Kripke frame with an evaluation over an arbitrary algebra offers a generalization of the concepts of necessity and possibility offer a rich setting to model and study graded and resource-sensitive notions from many different areas, including proof-theory, temporal and epistemic concepts, workflow in software applications, etc. While the first publications on modal many-valued logics can be traced back to the 90s [5, 6], it has been only in the latter years when a more systematic work has been developed, addressing the axiomatizability question over certain algebras of evaluation, characterization and study of model-theoretic notions analogous to the ones from the classical case, decidability and applicability issues, etc (see eg. [7], [3, 4], [1], [9], [8], [2]...).

An open problem was that of the axiomatization of the finitary companion of those deductive systems, starting from their definition based on Kripke models evaluated over FL_{ew} -algebras. In particular, they were not known axiomatizations for the modal logics arising from models with a crisp accessibility and using both \Box and \Diamond modalities, and evaluated locally at the standard Gödel, MV and product algebras. In this talk we will see that the global deduction over the classes of Kripke models over the standard MV and product algebras is not recursively enumerable, and so, they are not R.E axiomatizable.

Moreover, it was also a question whether for any of the above logics, the global deduction arises from the local one extended with the (unrestricted) necessity rule $N_{\Box}: \varphi \vdash \Box\varphi$. We will also see that this is not the case for a large family of algebras of evaluation, including the modal expansions of Lukasiewicz and Product Logics, in contrast to the modal logics studied up to now in the literature.

References

- [1] F. BOU, F. ESTEVA, L. GODO, AND R. RODRÍGUEZ, *On the minimum many-valued modal logic over a finite residuated lattice*, **Journal of Logic and Computation**, vol. 21 (2011), no. 5, pp. 739–790.
- [2] X. CAICEDO, G. METCALFE, R. RODRÍGUEZ, AND J. ROGGER, *A finite model property for Gödel modal logics*, **WOLLIC 2013, Lecture Notes in Computer Science**, (In L. Libkin, et.al eds.), vol. 8071, Springer (2013), pp. 226–237.
- [3] X. CAICEDO AND R. O. RODRIGUEZ, *Standard Gödel modal logics*, **Studia Logica**, vol. 94 (2010), no. 2, pp. 189–214.
- [4] X. CAICEDO AND R. O. RODRIGUEZ, *Bi-modal Gödel logic over $[0, 1]$ -valued kripke frames*, **Journal of Logic and Computation**, vol. 25 (2015), no. 1, pp. 37–55.
- [5] M. FITTING, *Many-valued modal logics*, **Fundamenta Informaticae**, vol. 15(1992), pp. 235–254.
- [6] M. FITTING, *Many-valued modal logics II*, **Fundamenta Informaticae**, vol. 17 (1992), pp. 55–73.
- [7] G. HANSOUL AND B. TEHEUX, *Extending Lukasiewicz logics with a modality: Algebraic approach to relational semantics*, **Studia Logica**, vol. 101 (2013), no. 3, pp. 505–545.
- [8] G. METCALFE AND N. OLIVETTI, *Towards a proof theory of Gödel modal logics*, **Logical Methods in Computer Science**, vol. 7 (2011), no. 2, 27.
- [9] A. VIDAL, F. ESTEVA, AND L. GODO, *On modal extensions of product fuzzy logic*, **Journal of Logic and Computation**, vol. 27(2017), no. 1, pp. 299–336.