Decidability of the Relational Syllogistic with Reordered Predicates

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Abstract

The *relational syllogistic* extends the *classical syllogistic* by introducing relational predicates to the language, so that, for example, the following sentences can be properly expressed:

Every student likes some philosophers: $(\forall S, \exists P)L$
Some students do not like some philosophers: $(\exists S, \exists P)\neg L$

The formalization in the examples above and below follows the Quantified Argument Calculus (Quarc), originally introduced in [1].

Relational syllogistic logics and their decidability have received several treatments in the literature. Some of those containing only unary and binary predicates were studied intensively in a series of works including [3]. But one may want the language to include n-ary predicates for all $n > 0$, so that we can further account for sentences like

Every philosopher gives some books to some students: $(\forall P, \exists B, \exists S)G$
Some philosophers do not give some books to any students: $(\exists P, \exists B, \forall S)\neg G$

Admittedly, all these sentences can be translated into the fluted fragment of first-order logic, and thus the decidability of such logics is guaranteed. However, one may still wonder how the problem can be treated directly, preferably with a concrete decision procedure. A work in a similar direction is [2], where the language contains predicates of all finite arities. But that system does not allow existential quantifiers to occur in the scope of universal ones, so the expressive power is limited.

In addition, one may also be interested in the results of adding reordered form of predicates, which enables us to capture sentences such as

Some philosophers are not liked by some students: $(\exists P, \exists S)\neg L^{2,1}$
Some books are given by every philosopher to some students: $(\exists B, \forall P, \exists S)G^{2,1,3}$

where we have $\forall x \forall y (L^{2,1}(y, x) \leftrightarrow L(x, y))$ and $\forall x \forall y \forall z (G^{2,1,3}(y, x, z) \leftrightarrow G(x, y, z))$. (Note that this feature is not covered by the fluted fragment.)

This paper is devoted to proving the decidability of the relational syllogistic where there are (reordered) predicates of all finite arities and no restriction is set on the scope of quantifiers. This is done in the following steps.

First, we present a formal language for the relational syllogistic in question, where we adopt the syntax and semantics of Quarc, and we notice that the result is the fragment of Quarc where anaphors and sentential operators are excluded.

Second, we formulate a tableau calculus which does not have "branching" rules, and we show that, apart from soundness and completeness, it has some other properties which are useful in the next step.

Third, with certain techniques for removing formulas that are inessential to the closure of tableaux, we show that

1. if a tableau is closed by a clash between two formulas with n-ary predicates for $n > 1$, they must be the descendants of two formulas (in the initial tableau) where the predicates have "unifiable" arguments;
2. the unifiability in 1 is determined by the closure of tableaux for specific sets of formulas involving only unary predicates.

And then we have the following result:

A finite set of formulas Γ effectively determines C*, a finite collection of finite sets of formulas containing no n-ary predicates for $n > 1$, such that Γ has a closed tableau iff for some Δ in* C*, Δ has a closed tableau.* (Some additional conditions need to be specified here when reordered predicates are in the language, but they will not affect the conclusion below.)

Hence, we have reduced the problem to one with only unary predicates, and as the tableau calculus is a decision procedure in that case, we conclude that the satisfiability of finite sets of formulas is decidable.

Finally, we present a syllogistic proof system for the logic (where no individual constants are used), and we show that its soundness and completeness are corollaries of certain propositions involved in the proof of the decidability.

References

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