On a Super-Classical System of Dialetheic Logic

Isaac Michael Hicks

June 2025

1 Proposal

I have developed a super-classical system of dialetheic logic. This system extends ST Logic with two non-classical connectives. These connectives can only be defined within a three-valued system and the non-classical inferences which they introduce are only valid when using an ST-consequence relation. They also form a functionally complete set. These connectives introduce non-classical, dialetheic inferences into ST Logic. The unary connective is referred to as *alteration* or "*alt-A*" (for alternative) while the binary connective is referred to as *mediation* or "*A mid B*" (for middle). I refer to this system as "Logic of Trivalence" (LT): the logic of *three-ness*. Provided below are their truth-tables:

$f\sim$	
1	i
i	0
0	1

Table 1: *Alteration*: *alt-A*

$f\#$	1	i	0
1	1	1	i
i	1	i	0
0	i	0	0

Table 2: *Mediation*: *A mid B*

$\Gamma \models_{ST} \varphi$ *iff*: for any valuation v, if $v(\psi) = 1$ for all $\psi \in \Gamma$, then $v(\varphi) = 1$ or $v(\varphi) = i$

Classical logic can be understood as one elaborate system of *duality*. There exists the dual truth-values of *false* and *true*. There exists the dual connectives of *conjunction* and *disjunction*. And there exists the dual quantifiers of *universal* and *particular*. Each of these truth-values, connectives, or quantifiers is then

defined in relation to their *dual* through *negation*. What becomes inexpressible are those concepts which lay *between* this duality. It is this space between duals - between *false* and *true*, between *conjunction* and *disjunction*, and between *universal* and *particular* - which motivates my logic. By reasoning *between* duals, my system is able to reason *beyond* classical logic. And yet, how might one begin to comprehend that which is beyond classical reasoning? The truth-table for *negation* within Kleene's three-valued logic provides the answer to this question. Within this truth-table, we find an element within our system which has no dual (or is its own dual). This *invariance under negation* is inconceivable within classical reasoning. By radicalizing this principle, we can construct an idempotent connective which also has no dual (or is its own dual). I refer to this connective as *mediation*. To *mediate* is to take the *middle*. This connective of *mediation* - like our third truth-value - is *invariant under negation*.

Within LT, we interpret our third truth-value as *both false and true*: *false* as a premise and yet *true* as a conclusion (and therefore *both*). Our definition of consequence is the mixed consequence relation of ST Logic, wherein *both false and true* is undesignated as a premise and yet designated as a conclusion. By taking the middle truth-value, *mediation* itself functions as *both* a *conjunction* and a *disjunction*: *conjunctive* as a premise and yet *disjunctive* as a conclusion. It exhibits *self-duality* while also satisfying the DeMorgan Laws. When interpreted using an ST-consequence relation, *mediation* logically satisfies all axioms of Boolean algebra. Our unary connective of *alteration* removes half of the truth-value from a proposition, with the condition that a *false* proposition becomes *true*. For this reason, triple *alteration* - like double *negation* - is eliminable. The non-classical inferences which can be proven using *alteration* and *mediation* are fundamentally dependent upon an ST-consequence relation. Because we can always find an interpretation wherein our premises are *true* and our conclusion is *both* - or our premises are *both* and our conclusion is *false* - these inferences are invalid when using the consequence relations of K3 or LP. The usability of *alteration* and *mediation* essentially depends upon an ST-consequence relation.

Taken together, *alteration* and *mediation* are functionally complete. For example, "*not-A*" is logically identical to "*alt-A mid alt-alt-A*". Similarly, neither *alteration* nor *mediation* are constructible using the connectives of either strong or weak Kleene. The connectives of Kleene lack functional completeness (therefore K3 has no tautologies). Because LT is an extension of ST Logic, it contains all classical inferences and tautologies. It also shares the same strong Kleene truth-tables for *negation*, *conjunction*, *disjunction*, and *implication*. However,

alteration and *mediation* add non-classical inferences. LT is therefore a super-classical logic. These are the basic, non-classical inferences which can be proven using our non-classical connectives. We use a tilde as "*alt*" and a hash as "*mid*": 1. *Alteration*: "$A \models \sim A$"; "$\sim\sim A \models A$"; "$A \models \sim\sim\sim A$"; and "$\sim\sim\sim A \models A$". 2. *Mediation*: "$A \models A \ \# \ B$"; "$A \ \# \ B \models B$"; "$B \models A \ \# \ B$"; "$A \ \# \ B \models A$"; "$A \ \# \ \neg A \models B$"; and "$\models A \ \# \ \neg A$". "$\neg(A \ \# \ B)$" is identical to "$\neg A \ \# \ \neg B$", while "$\neg(A \ \# \ \neg A)$" is identical to "$A \ \# \ \neg A$". Because "$A \ \# \ \neg A$" always valuates to *both*, it can *entail* any conclusion and be *entailed* by any premise. The inferences of *mediation* are exactly the same as AN Prior's connective *tonk*. Because ST-consequence is intransitive, *mediation* does not trivialize our logic.

The philosophy behind LT is dialetheic. Within the history of philosophy, the same duality exhibited within classical logic is also exhibited within philosophical debates. The possible answers to philosophical questions often split into two competing camps. There exists epistemological Empiricism and there exists epistemological Rationalism. There exists metaphysical Materialism and there exists metaphysical Idealism. And yet, within these debates, there often exists a third position which is irreducible to either camp. Instead, it acts as the *mediation* - as the *middle* - of these competing theories. Take for example epistemology. Kant's philosophy of Transcendental Idealism *mediates* the distinction between Empiricism and Rationalism. It exists in the middle of these two theories and is composed equally of both. As competing theories of knowledge, the *intersection* of Empiricism and Rationalism is underdetermined, while their *union* is overdetermined. However, their *mediation* produces a new theory of Transcendental Idealism. When interpreted using the basic inferences of *mediation*: If Empiricism E is *true*, then Transcendental Idealism $E \ \# \ R$ is *both false and true*. If Transcendental Idealism $E \ \# \ R$ is *both false and true*, then Rationalism R is *false*. And if Empiricism E is *true*, then Rationalism R is *false*. These types of arguments - these "dialetheic triads" - provide a justification for adding a non-trivializing, *tonk*-like connective to a dialetheic logic.

Through the use of non-classical connectives and ST-consequence, "Logic of Trivalence" has the capacity to model the logic between duals: the logic of a trinity. This trinitarian world both contains all classical inferences and tautologies - in the form of ST Logic - and yet also goes beyond it. The possibilities of a propositional system stronger than classical logic are essentially unknown. While classical, propositional logic has often been considered *maximal*, there exists a propositional *superset* to classical logic. This *superset* is the "Logic of Trivalence" (LT): the logic of *three-ness*. *Trivalence* is beyond structural *duality*.

2 References

1. Belnap, Nuel. "Tonk, Plonk and Plink." *Analysis* 22, no. 6 (1962): 130-134.

2. Cobreros, Pablo, Paul Egre, Ellie Ripley, and Robert van Rooij. "Inferences and Metainferences in ST." *Journal of Philosophical Logic* 49 (2020): 1057-1077.

3. Cobreros, Pablo, Paul Egre, Ellie Ripley, Robert van Rooij. "Tolerant, Classical, Strict." *Journal of Philosophical Logic* 41 (2012): 347-385.

4. Cook, Roy. "What's Wrong with Tonk?" *Journal of Philosophical Logic* 34, no. 2 (2005): 217-226.

5. Fjellstad, Andreas. "How a Semantics for Tonk Should Be." *Review of Symbolic Logic* 8, no. 3 (2015): 488-505.

6. Hegel, Georg Wilhelm Friedrich. *Science of Logic.* Cambridge: Cambridge University Press, 2010.

7. Jobe, William. "Functional Completeness and Canonical Forms in Many-Valued Logics." *Journal of Symbolic Logic* 27, no. 4 (1962): 409-422.

8. Kant, Immanuel. *Critique of Pure Reason.* Cambridge: Cambridge University Press, 1998.

9. Post, Emil L. "Introduction to a General Theory of Elementary Propositions." *American Journal of Mathematics* 43, no. 3 (1921): 163-185.

10. Priest, Graham. *Beyond the Limits of Thought.* Cambridge: Cambridge University Press, 1995.

11. Priest, Graham. *In Contradiction: A Study of the Transconsistent.* Oxford: Oxford University Press, 2006.

12. Priest, Graham. "Logic of Paradox." *Journal of Philosophical Logic* 8, no. 1 (1979): 219-241.

13. Prior, Arthur Norman. "Runabout Inference-Ticket." *Analysis* 21, no. 2 (1960): 38-39.