Constraining Meaning – Category Theoretic Perspectives in Semantics
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1 The Phenomena – Knowing *that*, but not *why*

Attitude verbs like *believe, know*, or *regret* embed a proposition expressing the content of the attitude. A sentence 'x believes that p', e.g., is usually depicted as true if all of x's doxastic alternatives are included in p (cf., e.g., Kratzer 2006). Following a Hintikkian (1969) possible world semantics, p is understood to be the set of worlds in which p is true. However, there are various scenarios in which this analysis of intensionality predicts solutions that aren't fine-grained enough or unintuitive.

(1) CONTEXT: Yoda has a logically well-trained mind and knows that p and $p \wedge (q \rightarrow p)$ are logically equivalent. Ahsoka, a young and stubborn padawan, is confident in believing p, but thinks that $p \wedge (q \rightarrow p)$ is saying something additional and is skeptic about it.
   a. Yoda believes that p and $p \wedge (q \rightarrow p)$.
   b. Ahsoka believes that p and $p \wedge (q \rightarrow p)$.

(2) CONTEXT: Theseus' starship (TS) flies through the galaxy and exchanges all of its parts during travel because of repair work. Watto, the junk dealer, quickly detects all repairs and does not recognize the identity of the ship that just arrived on Tatooine (T). The Jedi Obi-Wan can also see the repairs, but is simply happy about the return of Theseus and his crew. Thus, he recognizes the identity of the ship.
   a. Watto believes that Theseus' starship did not arrive on Tattoine.
   b. Obi-Wan believes that Theseus' starship arrived on Tatooine.

(3) CONTEXT: Anakin was once a famous Jedi and general in the Clone Wars. Later, he turned to the dark side and, using dark forces, grew even more powerful. For Rey, being a hero is to be powerful and fierce. For Kylo, being a hero is to be courageous and kindhearted.
   a. Rey believes that Anakin was a hero.
   b. Kylo believes that Anakin was a hero.

Although analytically, the logically equivalent beliefs in (1a.,b.) should have the same truth value, we know that beliefs are highly subjective and would like to grant Ahsoka her skepticism about $p \wedge (q \rightarrow p)$ while (inconsistently) believing p at the same time, thus allowing sentences a. and b. to have different truth values, despite the fact that the corresponding possible worlds are the same. Sentences a. and b. in (2) are a similar, yet not logical, but conceptual, version of the problem. The possible worlds analysis predicts $\{w \mid \text{TS arrived on T in } w\} \cap \{w \mid \text{TS did not arrive on T in } w\} = \emptyset$, as the embedded propositions in a. and b. together are logically inconsistent. Nevertheless, we are still inclined to grant both beliefs a label of truth to concede people different conceptualizations. This, of course, arises not only for inconsistent propositions a. and b., but also for consistent, or even superficially identical propositions as in (3), where the classical analysis predicts no difference, but knowing Rey's and Kylo's concepts, we would maybe like to discern them (hyper-)intensionally.
Moreover, it has been observed that some of these attitude verbs, which also allow for a Content DP, show different patterns of entailment, where (4) holds for *believe*, but not for *know* (cf. Djärv 2019, Elliott 2020).

(4) a. $\models$ b.
   a. Mary *attitude* {the story, the claim,...} that p.
   b. Mary *attitude* that p.

But it has not yet been noted that (4) only holds for *believe*-type verbs iff p has the same subject-matter (cf. Yablo 2014, example (1)), uses the same conceptualizations (examples (2) and (3)), and has the same structural reading (*de dicto* vs. *de re*) in both the DP- and the CP-version.

Previous work in semantics has generally focused on analysis based on type-driven selection and construction, operating on a definition of propositions as sets of possible worlds. Based on Lewis (1979), both Kratzer (2006) and Moulton (2009) include a logophoric compatibility of possible worlds with the belief of a subject in the meaning. As compatibility is based on possible worlds, the notion is too coarse-grained for the phenomena described above: the worlds are the same for (1a) and (1b), as well as for (3a) and (3b). Moreover, this notion of compatibility focuses on the content as an absolute unit only; therefore, it does not capture the structural relationship towards worlds, i.e., whether something in a belief is understood *de re* or *de dicto*. Interestingly, Djärv (2023) discusses the importance of sources when talking about beliefs, but does so only for combinatorial reasons and does not acknowledge how content must be anchored to a source in multiple dimensions.

Some finer-grained analyses are also available, depending on (a variation of) Truthmaker Semantics (cf. Yablo 2014, Fine 2017, or Moltmann 2020a, Moltmann 2020b), which, to some extent, capture the difference in (1), but not in (2) or (3). None of them discusses *de re* vs. *de dicto* readings for content nouns. Flexible scoping for capturing intensional effects as in (2) and (3) have been provided by Giorgolo & Asudeh (2011) and Asudeh & Giorgolo (2020), but in turn this approach misses the difference between (1a) and (1b), i.e. the distinction of subject-matter discussed by Yablo (2014).

The common link in formality between the phenomena and the related complex of problems has hitherto been overlooked – using inherently extensional functions to model inherently intensional phenomena. The extensionality of functions guarantees $f(2+2) = f(8-4)$, but does not reflect that the steps of the computation might differ. Extensional functions only tell us *that*, not *why*; but sometimes ways of being do indeed matter. Semantic functions understood on a set-theoretic basis exhibit the same shortcomings: Semantic components like *hero* or *Rey* are functionally consumed and out of each other's scope; thus, a relational analysis including the 'context' in a computation is not attainable extensionally. Truth-determining functions with 0 or 1 as outputs are the extremal point in that perspective, they do not reflect the computational steps, i.e. the ways in which a proposition is true. W.r.t. beliefs, we might, for example, want to have a flexible way to include the subject of an attitude into the scope of semantic computation of content whenever people have different concepts of things. A thorough semantic analysis should therefore reflect not only that something is true or false but also why. For obtaining the flexibility to include a contextual scope, on the basis of Król (2019) and Asudeh & Giorgolo (2020), we follow the considerations in Peruzzi (2006) of a set-free intensionality and argue that semantic analysis benefits from an epistemic notion of truth based on Category Theory rather than a realist one based on Set Theory. By abstracting from sets, we are able to formalize the relationship between semantic elements and their environment flexibly, which is necessary to formally reflect a (hyper-)intensional interpretation. We show that this perspective naturally leads to a constraint-based semantics rather than a type-driven functional one.

2 Abstracting away – Semantics without Sets

Semantics based on categories[1] rather than sets implies a shift of perspective: "A consequence of the Yoneda lemma implies that a fixed object in this category is determined up to unique isomorphism by the totality of its relationships to all other objects in the category." (Bradley, Terilla & Vlassopoulos 2022: 555) Therefore, the focus is more on how objects are related than on what objects are. As category theory is just an abstract description of structures such as sets (and everything based on that), we can formulate the same propositions without loss of expressivity. As a consequence, truth is conceptualized epistemically, since the Yoneda embedding relationally explains why, in a linguistic case, expressions have a certain meaning. Figure 1 shows a portion of the category of linguistic meaning, adapted from Asudeh & Giorgolo (2020: 24), and expanded for attitude verbs as *believe*.
The initial object 1 acts as an ontology, saying what there is via morphisms to other objects. Like product sets, there are product objects such as (e, b), where π_i is a projection morphism depicting the i-th component. Interestingly, in Category Theory, there is no difference between common objects (e-ntities, t-ruth values, b-eliefs) and function objects (e.g. $t^b \times b$, a function evaluating a belief's truth); everything is describable via morphisms between objects. To simplify the already quite dense figure, composite morphisms[2] and identity morphisms[3] are not depicted.

3 Constraining Meaning

Extensional vs. intensional contexts can be discerned by testing for robust reference. If both Yoda and the storm trooper punch Anakin, they punch the same entity and therefore both punch Darth Vader. In the context of *believe* though, their beliefs might differ because Yoda already knows about Anakin being Darth Vader, while the storm trooper lacks that information and only knows that Darth Vader is one of the Sith, but not Anakin.

(5) a. Yoda punches Anakin.
b. The storm trooper punches Anakin.

(6) a. Yoda believes that Anakin is a Sith. (*Anakin=Darth Vader*)
b. The storm trooper believes that Anakin is a Sith. (*Anakin≠Darth Vader*)

[1] Cf. Mac Lane (1998) or Riehl (2016) for a general introduction to category theory or Asudeh & Giorgolo (2020), Ch. 3, for a brief, but linguistically oriented overview.

[2] For every two morphisms $f : a \to b, g : b \to c$, there is a composite morphism $g \circ f : a \to c$.

[3] For every object c, there is an identity morphism $id_c : c \to c$. Linguistically, this corresponds to the equative copula as in *Anakin is Anakin*.

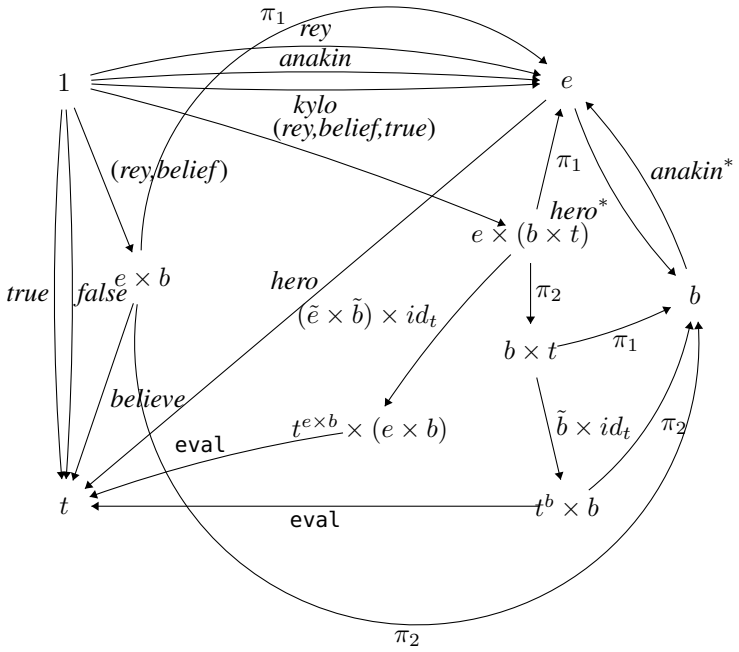

Figure 1: Linguistic category with beliefs

That difference can then be described with a simple constraint:

$$(a_1, x) \neq (a_2, x) \Rightarrow \pi_2 \circ (a_1, x) = \pi_2 \circ (a_2, x) \qquad \text{extensional case}$$
$$(a_1, x) \neq (a_2, x) \not\Rightarrow \pi_2 \circ (a_1, x) = \pi_2 \circ (a_2, x) \qquad \text{intensional case}$$

The constraint guarantees preserved identity of objects in the extensional case, while in intensional contexts that identity might be lost.[4] The constraints thus predict when to expect intensional effects that can be captured by the monadic mechanisms provided by Asudeh & Giorgolo (2020). But until now, we only know that potentially *anakin* $\neq$ *anakin** holds – what is special about *anakin** in contrast to *anakin*?

Conceptual meanings are found exactly in these contexts that are to be interpreted with respect to a subject potentially diverging from a standard extensional interpretation. E.g., the morphisms *anakin**, *hero** and their composite are expected to be interpreted according to Rey's perspective. The following constraint captures that:

$$c^* \circ h \circ (e, b, t) \Rightarrow c^* = c_e$$

c^* is the (potentially composite) morphism of a meaning term in an intensional context, h any morphism in between (here e.g.: $h = \pi_1 \circ \pi_2$), and (e, b, t) the tuple depicting e as the logical subject and b as the intensional object. c_e is abbreviated for the monadic interpretation of c, where c is interpreted with a perspectival index of the subject e. Thus, we can flexibly discern conceptual meaning differences for any scenario comparable to (2) or (3).

Subject-matter as defined by Yablo (2014) is naturally and constructively given by morphisms and their composite morphisms. A projectable SUBJECT-MATTER feature can collect the corresponding non-function morphisms of any daughter of a syntactic tree in a set, e.g. {*anakin,hero,hero* $\circ$ *anakin*} as possible subject-matters for the sentence *Anakin is a hero*. Therefore, we can easily differentiate the beliefs and respective truth evaluation in (1). Interestingly, this allows for a natural notion of parthood of content (cf. Fine 2013, Moltmann 2017) without any additional semantic 'machinery': The part p is believed by both Yoda and Ahsoka.

Constraining category theory-based meaning can offer flat descriptions of (hyper-)intensional contexts. Constraints for factivity, entailment patterns, and the distinction of *de re* vs. *de dicto* readings can be formulated in the manner developed above. The system of constraints naturally provides analysis applicable at the syntax-semantics interface in constraint-based approaches. It remains open if generative/transformation-based grammars could incorporate the system with the same ease. Moreover, it is worth noting that the Yoneda lemma can also show how transformational approaches and their standard formal semantics are essentially isomorphic to category theory-based descriptions.

[4] In Category Theory terms: Intensional contexts are usually described with monomorphisms.

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