Non-empty beliefs and belief revision using BSML

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Belief revision

Theories of belief revision describe how agents change their beliefs in light of new information. Dispositions to believe are formally represented as: $\text{B}^{\varphi}\psi$, which can be read as 'after learning φ the agent would believe ψ'. Alchourrón et al. (1985) (AGM) propose that agents' belief sets are closed under classical logical consequence. However, even though the sentence 'Shakespeare wrote Hamlet' (p) logically entails 'Shakespeare or Dickens wrote Hamlet' ($p \vee q$), believing the latter does not follow from believing the first ($\not\models \text{B}^{p}(p \vee q)$). Hence, many argue for a non-classical closure of belief sets (see e.g., Jago, 2014; Berto and Nolan, 2023; Hansson, 2022).

Berto (2019, 2022); Özgün and Berto (2021) propose to use a two-component semantics. The first component is standard modal logic with propositions identified with sets of possible worlds, and the second component is the ordered set of topics, which define the subject matter of each formula (determine what the formula is about). Hence, each sentence in the logical language denotes a proposition and is about some topic. In Berto (2019)'s framework, 'thought is hyperintensional,' since there are formulas φ and ψ denoting the same proposition, and hence equivalent ($\varphi \equiv \psi$), such that performing a belief update with those formulas yield different belief sets (for some χ: $\text{B}^{\varphi}\chi \not\equiv \text{B}^{\psi}\chi$), also correctly predicting that $\not\models \text{B}^{p}(p \vee q)$. This is possible since, in this framework, belief updates are topic-sensitive (hyperintensional) while the equivalence of non-modal formulas depends only on the first component: truth at the same possible worlds (intension). Another advantage of this system is that belief is not trivialized by inconsistent information; so $\not\models \text{B}^{\varphi\wedge\neg\varphi}\psi$.

I argue that Berto (2019)'s hyperintensional content ascription is neither necessary nor sufficient to capture intuitions about belief sets and belief revision using a logical framework. It is not necessary because the key observations can be captured by looking at the logic of information states with a basic cognitive principle: *neglect-zero* (Aloni, 2022), and it is not sufficient since it fails to account for the following intuitive entailments (Poss), (Taut 2) and (BSDA):

(Poss) $\text{B}^{\varphi}(\alpha \vee \beta) \models \langle\text{B}\rangle^{\varphi}\alpha$ (Taut 2) $\not\models \text{B}^{\varphi}(\varphi \vee \neg\varphi)$ (BSDA) $\text{B}^{\varphi\vee\psi}\chi \models \text{B}^{\varphi}\chi$

Sentences like (1) sound odd since it is hard to imagine a context in which Ann has no knowledge of how many children she has. They are only felicitous in cases like amnesia, where Ann truly does not remember. This is because disjunctions lead to possibility implications; if an agent comes to believe $p \vee q$ they consider both disjuncts live possibilities as in (Poss). A proposal based on relevance will have trouble accounting for this infelicity since both disjuncts have the same topic. Similar reasoning motivates (Taut 2). (BSDA) is motivated by the discussion of simplification of disjunctive antecedents (see, e.g., Santorio, 2020).

(1) #Ann believes that she has two-or-three children. $\text{B}(p \vee q)$ ~~$\text{B}p \vee \text{B}q$~~

Since $(p) = c(\neg p) = c(p \vee \neg p)$, then the opposite of (Taut 2) holds in (Berto, 2019). Moreover, (Poss) and (BSDA) fail because of the classical treatment of disjunction. Moreover, even though in Berto's system $\not\models \text{B}^{\varphi\wedge\neg\varphi}\psi$, the system fails to accurately represent the agent's belief state since it does not provide a direct link between beliefs and dispositions to believe (between $\text{B}\psi$ and $\text{B}^{\varphi}\psi$).[1]

[1] Berto (2022) proposes to model $\text{B}\varphi$ as $\text{B}^{\top}\varphi$, where $c(\top)$ is the set of topics that the agent

The logic of information states

I argue that hyperintensionality of thought can be instead explained by a distinction between the logic of truth-preservation and the logic of information states, understood classically as sets of possible worlds. Classical definition of belief by Hintikka (1962) treats it as a universal modality: $w \models B\varphi$ iff for all w' s.t. wRw': $w' \models \varphi$

Here, the image of R represents the *information state* of the agent at w. The agent believes everything which is true in (supported by) their information state. Classically, truth at an information state is reduced to truth at every point in that state. However, this is not the only possible logic of information states. For instance, Ciardelli et al. (2018a) argue for disjunction to be supported at a state whenever one of the disjuncts is true at every point. Aloni (2022) argues that a disjunction is supported whenever it is true at every point and each disjunct has a witness (see also: Väänänen, 2007a,b; Yang and Väänänen, 2016). Hence, in logics of information states, formulas are necessarily equivalent if they are supported at the same information states, which may not coincide with truth at the same worlds. Paraphrasing Ciardelli et al. (2018b): it is more difficult for formulas to be equivalent.

The proposal

Let $\mathcal{L}$: $\varphi := p \mid \text{NE} \mid \neg\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi \mid \text{B}\varphi \mid \text{B}^{\varphi}\psi$, let $\mathcal{F} = \langle W, \{R_\varphi \mid \varphi \in \mathcal{L}\}\rangle$ where we identify the set of all information states: $\mathcal{S} = P(W)$ and where for each formula $\varphi \in \mathcal{L}$ there is an accessibility relation $R_\varphi \subseteq W \times W$. A model $\mathcal{M} = \langle W, \{R_\varphi \mid \varphi \in \mathcal{L}\}, V\rangle$ such that $V : \mathit{Atom} \times W \mapsto \{0, 1\}$ assigns each propositional variable a truth value at any possible world. Observe that our models are simplifications of Berto (2019)'s since they do not contain the content-related components: $\mathcal{C}, \oplus$ and c.

To model belief revision, I will use the means of team semantics, where we evaluate formulas with respect to sets of assignments (sets of worlds) instead of single assignments (one world). I postulate that information states follow the following Bilateral State-Based Modal Logic proposed by Aloni (2022), who uses it to model assertions. I argue that belief ascriptions are strongly connected to claims about dispositions to assert. Let M be a model and let $s \in \mathcal{S}$ (for other clauses see Aloni (2022)):

$M, s \models p$ iff $\forall w \in s : V(w, p) = 1$ $\qquad$ $M, s \mathrel{=\!\!|} p$ iff $\forall w \in s : V(w, p) = 0$

$M, s \models \neg\varphi$ iff $M, s \mathrel{=\!\!|} \varphi$. $\qquad$ $M, s \mathrel{=\!\!|} \neg\varphi$ iff $M, s \models \varphi$.

$M, s \models \varphi \vee \psi$ iff $\exists t, t' : t \cup t' = s$ & $M, t \models \varphi$ & $M, t' \models \psi$.

$M, s \mathrel{=\!\!|} \varphi \vee \psi$ iff $M, s \mathrel{=\!\!|} \varphi$ and $M, s \mathrel{=\!\!|} \psi$.

These definitions define a logic equivalent to classical (modal) logic, and hence remain within the ‘classical toolkit’. The key non-classical component of this approach is the *non-emptiness atom* with support and anti-support conditions as follows:

$M, s \models \text{NE}$ iff $s \neq \varnothing$. $\qquad$ $M, s \mathrel{=\!\!|} \text{NE}$ iff $s = \varnothing$.

In this work, we will interpret sets of possible worlds as agents’ information states and say that agents believe all the sentences supported by their information state. Hence, let’s define the static belief operator as follows[2]:

has already grasped or the top element of the content lattice (Özgün and Berto, 2021). However, this causes the belief set to contain all contradictions whose topics were already grasped. Hence if $c(q) \leq c(\top)$ then if $\text{B}p \wedge \neg p$ then also $\text{B}q \wedge \neg q$ or in the revision terms: $\text{B}^{p \wedge \neg p \wedge \top} q \wedge \neg q$.

[2]For simplicity, I do not discuss the logic of truth, i.e., I assume that agents’ beliefs are completely determined by their information state. To connect truth and beliefs, say when $w \models \text{B}\varphi$, and model e.g., knowledge, we can add an accessibility relation, which determines the information state

$M, s \models \text{B}\varphi$ iff $M, s \models \varphi$. $M, s \dashv \text{B}\varphi$ iff $\exists t \subseteq s$: $t \neq \varnothing$ & $M, s \dashv \varphi$.

Following Berto (2019) I will use Lewis-style set selection function $f_\varphi : \mathcal{S} \mapsto \mathcal{S}$ which for each state selects the most plausible state: $f_\varphi(s) = \{v \mid wR_\varphi v \;\&\; w \in s\}$. The belief revision operator is a universal modality restricted to the acquired information:

$M, s \models \text{B}^\varphi\psi$ iff $M, f_\varphi(s) \models \psi$ $M, s \dashv \text{B}^\varphi\psi$ iff $\exists t \subseteq f_\varphi(s)$: $t \neq \varnothing$ & $M, t \dashv \psi$

Hence, belief revision with φ amounts to a static belief in an updated information state $f_\varphi(s)$. Observe that the rejection clauses give us dual modal operators $\langle\text{B}\rangle\varphi$ and $\langle\text{B}\rangle^\varphi\psi$, which correspond to 'considering possible that'.

Following Berto (2019), we will assume that the set selection function $f_\varphi(s)$ represents subjective dispositions to revise beliefs, according to a private plausibility ordering represented by the Lewisian sphere models. Hence, for every state s we have available a set of spheres surrounding it: $\S(s) = \{S_0^s, S_1^s, S_2^s \ldots\}$ where $S_0^s = s$ and following Lewis and Berto (2019) if $i \leq j$ then $S_i^s \subseteq S_j^s$. We assume Limit. Let $f_\varphi(s) = \varnothing$, if $|\varphi| = \{\varnothing\}$ or $|\varphi| = \varnothing$ otherwise, let $f_\varphi(s) = max(\{t \mid t \in |\varphi| \;\&\; t \subseteq S_i^s\})$ where S_i^s is the smallest sphere where this set is non-empty: since propositions in BSML are union-closed, we can easily prove that $f_\varphi(s)$ exists and is unique for any φ and s. Moreover, I propose that agents must have non-empty support for every part of what they believe. I follow Aloni (2022) and represent this using enrichment $[\cdot]^+$: adding the NE atom recursively as a conjunct to each subformula; for any connective $\otimes$: $[\varphi \otimes \psi]^+ = ([\varphi]^+ \otimes [\psi]^+) \wedge \text{NE}$.

Comparison with Berto (2019)

All main results from Berto (2019) regarding *Simplification*, *Adjunction*, *non-monotonicity*, *PEP*, *Cut* and *C-monotonicity* as well as facts about disjunction and tautologies, hold in the proposed system for enriched formulas[3]. *Sucess* ($\models \text{B}^\varphi\varphi$) holds for all *believable* formulas (i.e, for all $\varphi \not\equiv (\text{NE} \wedge \neg\text{NE})$, so $\text{B}\varphi$ is not a contradiction).

Unlike Berto (2019), this proposal allows for deriving possibility implications of disjunctive beliefs (Poss) as well as (Taut 2) and (BSDA). Additionally, this framework provides a clear link between beliefs and belief revision: disposition to belief is equivalent to belief at a revised information state. In particular, we predict that updates with information that agents already believe is a fixed point of the selection function, and hence do not extend the belief set. Moreover, we achieve this without assuming any topics or content of the formulas beyond the propositions they denote.

The downside of this proposal is that all contradictions are treated the same: even though they do not trivialize the belief state, they are not *believable* $\not\models \text{B}^{[\varphi\wedge\neg\varphi]^+}[\varphi]^+$. To resolve this issue, we may follow Berto (2019)'s suggestion and use impossible worlds. Observe that to provide an accurate representation of agent belief states when dealing with contradictions, content-based frameworks require a similar move.

Berto (2019) argues that $\{\text{B}^\varphi\chi, \varphi \equiv \psi\} \not\models \text{B}^\psi\chi$ and for other principles highlighting the difference between the one component equivalence (propositions) and equivalence under belief (propositions + contents). If we keep fixed that '$\equiv$' and '$\prec$' refer to the truth at the same possible worlds, these principles are satisfied in the proposed system. In this sense, the system is hyperintensional. However, if one like Ciardelli et al. (2018b) or Aloni (2022) argues that the logic of truth is also state-based, then the principles may cease to hold, leaving us with an improved intensional semantics.

at each w following Aloni (2022)'s definitions for modalities (however, see the last paragraph).

[3]Proofs and discussion will be available in the full contribution.

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