All else being equal—Projection as similarity over context sets

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This work presents a dynamic analysis of presupposition projection based on similarity relations between local and global contexts, drawing on accounts of counterfactual conditionals (Lewis 1973; Stalnaker 1978). The proposal addresses issues with presuppositions triggered in counterfactual contexts, where existing local-context-based accounts fail (Heim 1983, 1992; Schlenker 2009). The analysis is illustrated using (1a), where the negative prejacent *John quit smoking* entails that John does not currently smoke. Its additional inference, that John used to smoke (1b) may persist under entailment-cancelling operators, i.e. it may project.

(1) a. *Anna didn't quit smoking.* b. *Anna used to smoke.*

However, if the global context already entails that Anna never smoked, the inference (1b) does not arise, raising the question of how to predict when it does. Existing local-context-based accounts address this by assuming the presupposition can be accommodated in its *local context*, i.e., assumed to be true in a set of worlds where some embedded content holds. However, these accounts make specific assumptions about the relationship between local and global contexts, leading lead to faulty predictions when considering counterfactual contexts that logically need to be distinct from the global context. This work resolves this issue, deriving projection by requiring the global context to be similar to, rather than identical to, the local context. It also addresses the triggering problem by treating presuppositions as entailments (Abrusán 2011) and deriving projection by calculating similarity over partitions of the context set determined by the question under discussion (QUD). The analysis is implemented in a propositional version of CDRT, using propositional variables to model local and global context sets (AnderBois et al. 2015).

Counterfactual contexts. Existing analyses using local contexts (Heim 1983, 1992; Schlenker 2009) struggle with expressions in counterfactual contexts that presuppose something already known to be false (e.g., *Anna never smoked, so she didn't quit smoking;* or *I don't smoke, but if I quit smoking I would go crazy*). The accounts differ in how they determine local contexts but share two core assumptions: (i) presuppositions must hold in the local context of their trigger, and (ii) if the local and global context are identical, presuppositions must also hold globally (i.e., project). When presuppositions conflict with global information, the standard solution is local accommodation: the problematic inference is added locally, avoiding global contradiction. But this only works when it is still consistent with background information, creating issues when it is already known to be false. This problem is illustrated below using the propositional labels in (2) for the counterfactual prejacent of negation in (3) and Heim's update semantics (4).

(2) a. $W := \{w_{su}, w_s, w_u, w_\emptyset\}$
b. $\neg s := \{w_u, w_\emptyset\}$ (A doesn't smoke)
c. $u := \{w_{su}, w_u\}$ (A used to smoke)

(3) *Anna didn't quit smoking.*
a. $c_0 + {\sim}(\partial u \wedge \neg s) = c - c_1$
b. $c_1 = (c_0 + \partial u) + \neg s$

(4) Heimian update semantics:
a. $c + p = c \cap p$
b. $c + \partial p = c$ if $c \subseteq p$, otherwise undefined
c. $c + {\sim}p = c - c_1$, where $c_1 = c + p$
d. $c + (p \wedge q) = (c + p) + q$
e. *Anna quit smoking:* $(\partial u \wedge \neg s)$

(3b) is defined only if the presupposition u holds in the input c_0. If u is false in $c_0 = \{w_s, w_\emptyset\}$, Heim assumes local accommodation. But since the local context c_1 derives from c_0, accommodating a globally false presupposition is contradictory ($c_0 \cap u = \emptyset$). Local accommodation of the presupposition triggered by *Anna quit smoking* in a context where it is known that Anna never smoked, thus either forces contradictions or requires empty local contexts. While assuming empty local contexts may solve simple cases, we need a general approach for reasoning over counterfactual presuppositions. E.g., *It's not true that Anna would be moody if she quit smoking,* can be informative even if Anna never smoked, presenting a problem for Heim's (1983) dynamic negation.

The issue is more general: Schlenker 2009 also treats local contexts under negation as identical to the global context, predicting that (3) is inconsistent in c_0. For counterfactual conditionals, both accounts make faulty predictions. Heim 1992 (pp. 204 f.) assumes that the local context is a superset of the global context, again enforcing global consistency for embedded presuppositions. Schlenker 2009 predicts that the local context is the entire domain W allowing only presuppositions that hold throughout W (see Mackay 2023). Heim 1992 identified a related issue for her analysis of non-veridical doxastic attitudes (e.g., English *believe*), which dissociates local from global contexts, and evaluates embedded presuppositions against the attitude holder's doxastic state, incorrectly predicting no projection. In sum, existing analyses either predict inconsistency or must stipulate that local contexts are somehow independent from the global context, in ways that undermine a unified theory of presupposition projection.

A similarity-based analysis. The core innovation here is to loosen the requirement that local contexts must be strictly identical to (or subsets of) the global context. Analyses of counterfactual conditionals (following Lewis 1973) use similarity over worlds to derive that local contexts are like the global context, except the embedded content is true. To derive projection, we assume the opposite: the global context is similar to whatever local context results from embedding a presupposition in negation, a conditional, or another entailment-cancelling operator. Concretely, following Abrusán 2011, I treat the presupposition in (1) as an entailment of the negative prejacent (5a), so that the local context of the prejacent is $\{w_u\}$, where all its entailments hold. When this propostion is negated (5b), we update the global context to satisfy the main assertion, and in addition, contain only the worlds most similar to the local context $\{w_u\}$. Put differently: We define dynamic negation (6a) so that, given the local context $c_1 = (u \wedge \neg s)$ the global output context is the set of worlds that is the subset of $c - c_1$ that is closest to c_1 (6b).

(5) a. *Anna quit smoking* $\rightsquigarrow (u \wedge \neg s)$, i.e., $\{w_u\}$
b. *Anna didn't quit smoking.* $\rightsquigarrow \sim(u \wedge \neg s)$

(6) a. **Update rule**: $c + \sim p = Sim_{c_1}(c - c_1)$, where $c_1 = p$
b. **Similarity**: $Sim_c(c') := \{w \mid \exists w' \in c(w \in sim_{w'}(c'))\}$

To formalize "closest worlds," we measure the distance between worlds w, w' by counting the atomic propositions (wt constants) assigning distinct truth values to w, w' (7a). Using a world w as reference point, we can define an equivalence relation (7b), which holds of two worlds w', w'', iff they are equidistant from w. For w_u, the equivalence classes on W (shown in Figure 1) are: $\{w_u\}$ ($d_{w_u} = 0$), $\{w_0, w_{su}\}$ ($d_{w_u} = 1$), and $\{w_s\}$ ($d_{w_u} = 2$) (based on Lewis 1973). The relation $sim_w(p)$ (7c) selects the equivalence class containing the w_u-closest worlds found in p. For instance, if $p = W$ includes w_u, this will be $\{w_u\}$, with $d_{w_u} = 0$. For $p = \{w_s, w_\emptyset\}$ which does not include w_u, this is $w_\emptyset$, the subset of p with $d_{w_u} = 1$.

(7) a. $d_w(w') := |\lambda p \in \mathbf{Const}_{wt}.p(w) \neq p(w')|$
b. $\sim_w = \lambda w'.\lambda w''.d_w(w') = d_w(w'')$
c. $sim_{w_u}(p) := \{w' \in p \mid \forall w'' \in p(d_{w_u}(w') \leq d_{w_u}(w''))\}$

The context update in (6a) ($Sim_{c_1}(c - c_1)$), therefore, selects those worlds in $c - c_1$ that lie in the minimal distance class for at least some world in c_1. This eliminates any world where the negative prejacent $\{w_u\}$ holds or that is too dissimilar from w_u. Consequently, when the presupposition u is contradicted by the input context $c_0 = \{w_s, w_\emptyset\}$, the resulting set is $\{w_\emptyset\}$:

(8) a. $c_0 +$ *Anna didn't quit smoking.* $= Sim_{c_1}(c - c_1)$, where $c_1 = (u \wedge \neg s) = \{w_u\}$
b. for $c_0 = \{w_s, w_\emptyset\}$: $Sim_{\{w_u\}}(\{w_s, w_\emptyset\} - \{w_u\}) = Sim_{\{w_u\}}(\{w_s, w_\emptyset\}) = \{w_\emptyset\}$

This analysis handles an utterance like *Anna didn't use to smoke, so she didn't quit smoking* without contradiction. Even if the presupposition is globally false, a local context can still invoke a counterfactual scenario where it is true. The effect of projection on the global context is calculated via similarity rather than rigid containment. The resulting output is $\{w_\emptyset\}$, eliminating w_s as too dissimilar to the local context. This inference, that Anna has not just now taken up smoking, aligns with intuitive judgments, though further research is needed to systematically assess the predictions of this similarity-based account.

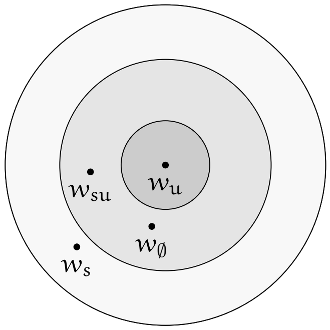

Figure 1:
w_u-distance classes over W

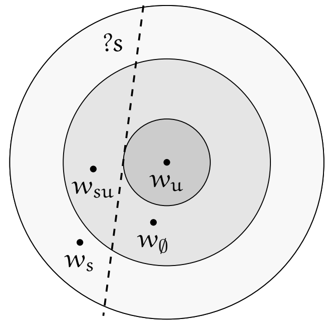

Figure 2: w_u-distance classes and partition ?s over W

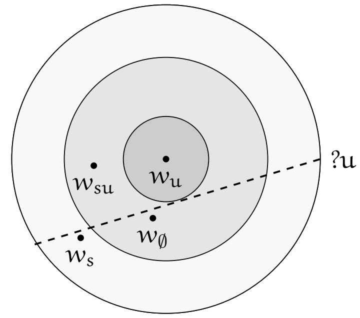

Figure 3: w_u-distance classes and partition ?u over W

QUD-sensitive triggering. A second component of this proposal addresses the triggering problem: not all entailments behave like presuppositions. Following Abrusán 2011 and Simons et al. 2017, we derive presuppositions as entailments are not at-issue relative to the QUD. This is necessary considering the case where (8) updates a context $c_0 = W$ that is undecided wrt the relevant propositions (9). Here, the output is $\{w_{su}, w_\emptyset\}$, predicting that u does not project—unlike existing accounts of projection.

(9) for $c_0 = \{w_{su}, w_s, w_u, w_\emptyset\}$: $\mathrm{Sim}_{\{w_u\}}(\{w_{su}, w_s, w_u, w_\emptyset\} - \{w_u\}) =$
$\mathrm{Sim}_{\{w_u\}}(\{w_{su}, w_s, w_\emptyset\}) = \{w_{su}, w_\emptyset\}$

Depending on the preceding context, either one of these worlds may be eliminated by the update. In (10), u projects, so the combined inferences $(s \wedge u)$ characterize the output context $\{w_{su}\}$. In contrast, (11) is interpreted by some speakers as $(\neg u \wedge \neg s)$, with the output $\{w_\emptyset\}$.

(10) Q: *Does Anna smoke?* A: *She didn't quit.*
a. suggests answer: s (yes, A smokes)
b. projects: u (A used to smoke)

(11) Q: *Did Anna smoke in the past?* A: *She didn't quit.*
a. suggests answer: $\neg u$ (no, A didn't smoke in the past)
b. projects: $\neg s$ (A doesn't smoke)

While the empirical the empirical status of the inference (11b) remains debated, inviting further study, this work provides a proof of concept for systematically deriving such inferences. We model the QUD as partitioning the context set (Ginzburg 1996; Roberts 1996) and calculate similarity to the local context for each QUD-partition. The partitions invoked in (10) and (11) are shown in (12), and the result of selecting the worlds most similar to the local context for each partition is given in (13). The interaction between QUD-partitions and w_u-distance is graphically illustrated in Figures 2 and 3.

(12) QUD-partitions:
a. $?s = \{\{w_s, w_{su}\}, \{w_\emptyset, w_u\}\}$
b. $?u = \{\{w_u, w_{su}\}, \{w_\emptyset, w_s\}\}$

(13) Applying similarity to partitions:
a. $\{\mathrm{Sim}_{\{w_u\}}(c) \mid c \in ?s\} = \{\{w_{su}\}, \{w_u\}\}$
b. $\{\mathrm{Sim}_{\{w_u\}}(c) \mid c \in ?u\} = \{\{w_u\}, \{w_\emptyset\}\}$

(14) Assertion eliminating answers:
a. $= \{\{w_{su}\}\}$
b. $= \{\{w_\emptyset\}\}$

Applying the projection mechanism to the QUD-partitions for ?s, from (10), yields the reduced partitions in (13a), all of which entail u (*Anna used to smoke*), correctly predicting projection. We assume that an inference projects if it is entailed by every partition of the context. The approach, thus, aligns the analysis of lexical presuppositions with Abusch's (2010) account of presupposition triggering from alternative sets (e.g., from focus, questions). Finally, we assume that assertion discards QUD-answers contradicted by the assertion; in combination with (14a), this derives the inferences outlined in (10). By contrast, ?u from (11), yields the partitions in (13b), where u is not entailed throughout, and thus does not project. However, $\neg s$ is entailed across the partitioned context set, matching the projective inference reported for (11).

Formal account. Finally, I show how the analysis is formalized in a propositional version of CDRT (Muskens 1996; AnderBois et al. 2015), where discourse states (sets of variable assignments) are updated distributively. Updates (15) are relations between input assignment g and output h, and introduce a propositional discourse referent ϕ and a series of conditions.

(15) $[\![[\phi \mid C_1, \ldots C_n]]\!]^{M,} = \{h \mid \exists p \subseteq D_w : h = g[\phi \mapsto p], [\![C_1 \wedge \cdots \wedge C_n]\!]^{M,h} = 1\}$

Discourse states store a designated propositional variable ϕ_{CS} representing the current context set, and a variable ϕ_{PS}, to model a projected set of possible future common grounds, storing the proposal made by an assertion (based on Farkas and Bruce 2010), and ϕ_{QUD} for QUD-partitions of the context set. The following illustrates the basic mechanism for questions and assertions, then shows how the projection of the non-at-issue presupposition (u) of (1) is derived when u is consistent with the context set. I then derive that there is no projection when the context entails that u is false.

The question *Does Anna smoke?* (16) introduces a QUD-partition of the context set and stores QUD-alternatives as possible values of ϕ_{QUD}. This is done by reintroducing the dref for ϕ_{QUD} to allow for multiple possible values, which are restricted to being subsets of ϕ_{CS} entailing either s or $\neg s$. The update also adds possible answers to the projected set. For an initial singleton discourse state $\{g\}$, s.t., $g(\phi_{CS}) = g(\phi_{QUD}) = g(\phi_{PS}) = U$, the output is $\{h_1, h_2\}$, assigning variables as in Figure 4.

(16) $?s := [\phi_{QUD} \mid \phi_{QUD} \subseteq \phi_{cs},\ \phi_{QUD} \subseteq s \vee \phi_{QUD} \subseteq \neg s];\ [\phi_{PS} \mid \phi_{PS} = \phi_{QUD}]$

	ϕ_{cs}	ϕ_{QUD}	ϕ_{PS}
g :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_{su}, w_s, w_u, w_\emptyset\}$

$\Rightarrow$

	ϕ_{cs}	ϕ_{QUD}	ϕ_{PS}
h_1 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_{su}, w_s\}$	$\{w_{su}, w_s\}$
h_2 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_u, w_\emptyset\}$	$\{w_u, w_\emptyset\}$

Figure 4: Update with $?s$ for the input context $\{g\}$, where $g(\phi_{CS}) = g(\phi_{QUD}) = g(\phi_{PS}) = \{w_{su}, w_s, w_u, w_\emptyset\}$

Assertions are proposals to update the context set, but directly change only the projected set (Farkas and Bruce 2010). We update ϕ_{PS} to include only the QUD-answer(s) entailed by the assertion. Asserting *Anna smokes* (17) after (16) has the output state illustrated in Figure 5. If ϕ_{PS} has only one non-empty value, the assertion can be accepted by setting: $\phi_C = \phi_{PS}, \phi_C \neq \emptyset$.

(17) $\textsc{assert}(s) := [\phi_1 \mid \phi_1 = s]; [\phi_{PS} \mid \phi_{PS} = \phi_{QUD} \vee \phi_{PS} = \emptyset,\ \phi_{PS} \subseteq \phi_1]$

	ϕ_{cs}	ϕ_{QUD}	ϕ_{PS}
h_1 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_{su}, w_s\}$	$\{w_{su}, w_s\}$
h_2 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_u, w_\emptyset\}$	$\{w_u, w_\emptyset\}$

$\Rightarrow$

	ϕ_{cs}	ϕ_{QUD}	ϕ_{PS}	ϕ_1
i_1 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_{su}, w_s\}$	$\{w_{su}, w_s\}$	$\{w_{su}, w_s\}$
i_2 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_u, w_\emptyset\}$	$\emptyset$	$\{w_{su}, w_s\}$

Figure 5: Update when asserting s after asking whether $?s$ in g.

Projection. Following the question (16), we could instead assert *Anna quit smoking* ($u \wedge \neg s$) (18).

(18) $\textsc{assert}(\textsc{not}(u \wedge \neg s)) :=$

a. $[\phi_1, \phi_2 \mid \phi_1 = \overline{\phi_2},\ \phi_2 = (u \wedge \neg s)];$

b. $[\phi_{QUD} \mid \phi_{QUD} \subseteq \phi_{PS}, \mathtt{Sim}(\phi_{QUD}, \phi_2)];$

c. $[\phi_{PS} \mid \phi_{PS} = \phi_{QUD} \vee \phi_{PS} = \emptyset,\ \phi_{PS} \subseteq \phi_1]$

As shown in Figure 6, (18a) contributes a dref ϕ_2 storing the local context $\{w_u\}$ where $u \wedge \neg s$ is true (Stone and Hardt 1999; Krifka 2013). (18a) also introduces a dref ϕ_1 for $\{w_{su}, w_s, w_\emptyset\}$, introduced by the main clause as the complement of ϕ_2, implementing the semantics of negation. (18b) derives projection of *Anna used to smoke* (u) by updating ϕ_{QUD} s.t., for each QUD-partition, only the worlds most similar to the local context are retained. This eliminates any world $w \in \phi_{QUD}$ that is less similar to $\{w_u\}$ than another world in the same partition (details below). For $h_1(\phi_{QUD}) = \{w_{su}, w_s\}$, we get $i_1(\phi_{QUD}) = \{w_{su}\}$, and for $h_2(\phi_{QUD}) = \{w_u, w_\emptyset\}$, $i_2(\phi_{QUD}) = \{w_u\}$. As a result, all possible answers to the QUD now entail u, deriving projection. Finally, (18c) updates ϕ_{PS} to contain only answers to the QUD entailed by the assertion.

	ϕ_{cs}	ϕ_{QUD}	ϕ_{PS}
h_1 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_{su}, w_s\}$	$\{w_{su}, w_s\}$
h_2 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_u, w_\emptyset\}$	$\{w_u, w_\emptyset\}$

$\Rightarrow$

	ϕ_{cs}	ϕ_1	ϕ_2	ϕ_{QUD}	ϕ_{PS}
i_1 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_{su}, w_s, w_\emptyset\}$	$\{w_u\}$	$\{w_{su}\}$	$\{w_{su}\}$
i_2 :	$\{w_{su}, w_s, w_u, w_\emptyset\}$	$\{w_{su}, w_s, w_\emptyset\}$	$\{w_u\}$	$\{w_u\}$	$\emptyset$

Figure 6: Update when asserting $\neg(u \wedge \neg s)$ after asking whether $?s$ in g.

Conclusion / Outlook. This work proposes a similarity-based dynamic semantics, where local and global contexts are related via similarity rather than identity or containment, thereby addressing presuppositions in counterfactual environments without creating contradictions. The QUD-sensitive projection mechanism also addresses the triggering problem: presuppositions arise as entailments that are not at-issue relative to the QUD. Viewing triggering, accommodation, and projection through this unified, QUD-driven mechanism invites broader applications to discourse contexts where multiple triggers, questions, and background assumptions interact in more complex ways. The full paper shows details how this approach predicts no projection in contexts like *Why is Anna chewing her pencil? (Well,) she didn't quit smoking* (based on Geurts 1994), or where the presupposition has been explicitly contradicted. Beyond negation, the paper shows how the proposal extends to conditionals, epistemic modals, and other entailment-canceling operators, thus surpassing the empirical reach of both existing formal semantic and RSA-style approaches.

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