# Group Epistemics, Co-algebraically

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This contribution concerns group epistemic logics where group epistemic attitudes are modelled using modalities with algebraic labels used to refer to groups. The framework builds on epistemic logic with names of [6], and concerns logics for structured intensional groups considered in [2]. It is based on a multimodal expansion of the logic known as bi-modal monotone logic [10]. Our goal is to provide an abstract, coalgebraic account of bi-modal monotone logic and epistemic logics based on it.

Our work is motivated by lifting one of the common assumptions of multi-agent epistemic logic: that groups of agents are given *extensionally* as sets of agents. As a consequence, group membership is common knowledge to all agents, and any change in membership implies a corresponding change in the identity of a group. This is not necessarily how we think of groups: we routinely reason in various contexts without knowing the extensions of groups - we might, for example, refer to groups such as "bot accounts", "democrats", or "correct processes" - and we are not satisfied with reducing groups to their extensions either, since they can clearly change over the state space of a system or over possible states of the world. Epistemic logics of intensional groups lift the above assumptions by seeing groups as given to us *intensionally* by a common property that can change its extension from world to world—as, e.g., being a "consistent subgroup" does. In their seminal work [6, 5], Grove and Halpern introduced a multi-agent epistemic logic in which groups are labeled using a set of abstract *names* and the naming relationship can vary from world to world. The language originally contains two types of modalities: $E_n\varphi$ means "everyone named $n$ knows that $\varphi$ ", and $S_n\varphi$ means "someone named $n$ knows that $\varphi$ "; they further consider a natural extension of the basic framework where names are replaced by formulas expressing *structured* group-defining concepts. Epistemic logic with names of [6] was in a sense seminal to the development of term-modal logic introduced by [4], and can be seen as its simple decidable fragment. (a closely related language of implicitly quantified modal logic was studied in [11]). In [1], we studied expansions of epistemic logic with names with non-rigid versions of common and distributed knowledge, derived in the usual way from $E_n\varphi$ and $S_n\varphi$ modalities respectively. Alternatively, Humml and Schröder [9] generalize Grove and Halpern's approach to structured names represented by formulas defining group membership, including, for example, formulas of the description logic ALC. Their abstract group epistemic logic AGEL contains a common knowledge modality as the only modality and, unlike in [1, 6], their group names are rigid.

In [2], we adopted the perspective that (i) both "everyone labeled $a$ knows" and "someone labeled $a$ knows" modalities form a minimal epistemic language for group knowledge where groups are understood intensionally, and (ii) that their labels reflect their structured nature. We used languages built on top of classical propositional language containing modalities $[a], \langle a]$ indexed by elements of an algebra of a given signature of interest, we set up a general relational semantics involving an algebra of group labels to index (sets of) relations in each world, and shown how some related logics can be modelled in such a way. The minimal logic (the logic for the empty signature of labels, i.e. the one with set of names)

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coincides with multimodal expansion of a logic known as bi-modal monotone logic [10], and traces back to Brown's Logic of ability [3].

Although monotone modal logic (which coincides with the "someone labeled *a* knows" fragment) is well known and investigated coalgebraically, the combination of monotone and normal operators involved in the case of bi-modal monotone logic has not yet received the same kind of attention. Following previous work by Hansen et al. on monotone modal logic and coalgebraic dynamic logic [7, 8], we give a coalgebraic account of the logics considered in [2] in two different ways: first way relies on coalgebras of a subfunctor of the product of monotone neighborhood and filter monads, capturing the interaction of a monotone and a normal modality. We show how bi-modal monotone logic arises as a coalgebraic logic for this functor. Second way uses double covariant powerset functor, which is known not to be a monad. The second approach is closer to [2], but the language is not expressive for bisimulations.

We further concentrate on linking two-sorted algebras involving propositions and group labels/types of knowledge with appropriate neighborhood coalgebraic semantics, in terms of an appropriate topological or discrete duality. This can hopefully be further applied, e.g. to obtain a definability theorem or to design a multi-type proof theory for basic logic. We also discuss some particular examples of algebraic signatures that give rise to interesting and useful variants of group knowledge, such as distributed or common knowledge.

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