

Damien Fleury and Lucia M. Tovenà

# Extending the benchmark for the monotonicity of superlatives

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- Bumford and Sharvit (2022) claimed to give a benchmark for testing the monotonicity properties of definite superlatives.
  - (1) The highest grade John (ever) got was kind of low. (absolute)
  - (2) John got the highest grade. (absolute/relative)
  - (3) Mary sang the loudest that anyone sang. (relative)
- Their discussion does not include approaches to modal superlatives though, e.g. (4).
  - (4) Mary was the nicest possible. (modal)
- This talk takes up from there. We cross examine the monotonicity of the modal superlative operators recently proposed by Loccioni (2018, 2019)—and Romero (2013) she relies on—and Tovena and Fleury (2023, 2024).
- They pass the test, but the matter goes beyond that. The empirical issue of the licensing power of superlatives is back on the agenda (see Hoeksema 2012 i.a.), as well as the issue of the characterisation of their logical properties.

# Outline

- ① Highlights from Bumford and Sharvit (2022)
- ② Loccioni (2019) and Romero (2013)
- ③ Tovina and Fleury (2024)
- ④ Discussion

## Highlights from Bumford and Sharvit (2022)

# $\text{est}^{\text{vF}}$ from von Fintel (1999) and Gajewski (2010)

- $\llbracket \text{est}^{\text{vF}} \rrbracket = \lambda R \lambda P \lambda x [\exists d [\{x\} = \{x' : P(x') \wedge R(d)(x')\}]]$   
Presup.  $P(x) \wedge \exists d [R(d)(x)]$
- $\text{est}^{\text{vF}}$  applies to a degree predicate  $R$  of type  $\langle d, et \rangle$  that relates individuals and degrees, a property of entities  $P$ , and an entity  $x$
- It states that  $x$  is the entity who attains the highest degree (via  $R$ ) among the entities who are  $P$ .

(5) a. The highest grade John (ever) got

b. [the [high  $\text{est}^{\text{vF}}$ ] [grade [2 [John (ever) got  $t_2$ ]]]]

$$R = \llbracket \text{high} \rrbracket = \lambda d \lambda x' [ht(x') \geq d]$$

$$P = \llbracket \text{gradeJohn(ever)got} \rrbracket = \lambda x' [grade(x') \wedge got(x')(j)]$$

c.  $\iota x [\exists d [\{x\} = \{x' : grade(x') \wedge got(x')(j) \wedge ht(x') \geq d\}]]$

- When combined with  $R$ , the operator has the right monotonicity properties to license NPIs.
- However, it is devised for the absolute reading of the superlative, and Bumford and Sharvit do not succeed in adapting it to the relative reading.

# est<sup>H1</sup> from Heim (1999)

- $\llbracket \text{est}^{\text{H1}} \rrbracket = \lambda R \lambda x [\exists d [\{x\} = \{x' : R(d)(x')\}]]$   
Presup.  $\exists d [R(d)(x)]$
- est<sup>H1</sup> applies to a predicate of degrees  $R$  of type  $\langle d, et \rangle$  and to an entity  $x$ .
- Operator est<sup>H1</sup> collapses in predicate  $R$  the information that an entity both reaches a degree  $d$  and is in the extension of some specific property. It indicates that  $x$  is the entity that reaches the highest degree (via  $R$ ).

(6) John got the highest grade.

- a. John got a perfect score (the grade higher than any other grade).

[John got [the [est<sup>H1</sup>] [1 [high- $t_1$  grade]]]]

$\text{got}_w(\iota x [\exists d [\{x\} = \{x' : \text{grade}_w(x') \wedge \text{ht}_w(x') \geq d\}]])(j)$

- b. John got a higher grade than anyone else did.

[John [est<sup>H1</sup>] [1 [got *the* high- $t$  1 grade]]]]

$\exists d [\{j\} = \{z : \exists y [\text{grade}_w(y) \wedge \text{got}_w(y)(z) \wedge \text{ht}_w(y) \geq d]\}]$

- Such an operator allows for both absolute and relative readings of the superlative, but does not have the right monotonicity properties.

# est<sup>H2</sup> from Heim

- $\llbracket \text{est}^{\text{H2}} \rrbracket = \lambda C \lambda P [\exists d [\{P\} = \{Q \in C : Q(w)(d)\}]]$   
Presup.  $P \in C \wedge \forall Q \in C [\exists d [Q(w)(d)]]$
- $\text{est}^{\text{H2}}$  applies to a collection  $C$  of (intensional) degree properties of type  $\langle s, dt \rangle$  and to an element  $P$  of this collection.
- It states that  $P$  is the property associated with the greatest degree in the world of evaluation, among the properties of collection  $C$ .
- Such  $C$  plays the role of comparison class and can be expressed as a sort of relative clause in the case of a construction with antecedent-contained deletion (ACD).

(7) a. Mary sang the loudest that anyone sang.

b.  $\llbracket [\text{est} [1 [\text{anyone sang } t_1\text{-loud}]]] [1 [\text{Mary sang } t_1\text{-loud}]] \rrbracket$

c.  $\exists d [\{ \lambda w' \lambda d [\text{Mary sang}_{w'} d\text{-loud}] \} = \{ Q \in C : Q(w)(d) \}]$

$$C = \{ \lambda w' \lambda d [x \text{ sang}_{w'} d\text{-loud} : \text{person}_w(x)] \}$$

$$P = \lambda w' \lambda d [\text{Mary sang}_{w'} d\text{-loud}]$$

- This operator  $\text{est}^{\text{H2}}$  has the right monotonicity properties, but Bumford and Sharvit try without success to adapt it to allow for an absolute reading in such a construction.

## Strawson entailment

- To check the monotonicity properties of an operator, we use the Strawson entailment, as presented in Bumford and Sharvit (2022):
  - ▶ Polymorphic Strawson entailment :
 

$X \Rightarrow^{ST} Y$  iff either (i) or (ii) is true.

    - X and Y are truth values, and  $X \rightarrow Y$ .
    - X and Y are functions of type  $(\sigma, \tau)$ , and  $X(a) \Rightarrow^{ST} Y(a)$  for every a in the domain of both X and Y.
  - ▶ A function  $f$  is Strawson downward-entailing (SDE)
 

iff  $f(Q) \Rightarrow^{ST} f(P)$  whenever  $f(P)$  and  $f(Q)$  are both defined and  $P \Rightarrow^{ST} Q$ .
  - ▶ A function  $f$  is Strawson upward-entailing (SUE)
 

iff  $f(P) \Rightarrow^{ST} f(Q)$  whenever  $f(P)$  and  $f(Q)$  are both defined and  $P \Rightarrow^{ST} Q$ .
  - ▶ An NPI is licensed only if it occurs in the scope of an operator  $\alpha$  such that  $\llbracket \alpha \rrbracket^{w,g}$  is SDE and not SUE, for any  $w$  and  $g$ . Sometimes  $\alpha$  is called the NPI licenser.



## Loccioni (2019) and Romero (2013)

## Romero (2013)

- (8) a. John climbed the highest possible mountain. (attributive)  
 b. [-est [1 possible <for John(/him) to climb A  $t_1$ -high mountain>]] [2 John climbed A  $t_2$ -high mountain]
- Romero collects the set of possible degrees ([1 possible <for John(/him) to climb A  $t_1$ -high mountain>]), and the degree property of the mountain climbed by John in the actual world ([2 John climbed A  $t_2$ -high mountain]).
  - But Romero wants to use an operator à la Heim, that is she wants to compare a degree property to a set of degree properties.
  - For that, she turns the set of possible degrees into a set of degree properties, using the following operator :
- $$\text{SHIFT}_{\langle d, t \rangle \rightarrow \langle dt, t \rangle} = \lambda D_{\langle dt \rangle} \lambda D'_{\langle dt \rangle} [\exists d' [D(d') \wedge D' = \lambda d'' [d'' \leq d']]]$$

## Romero (2013)

- Then Romero can use the Heim-like operator in (9).

$$(9) \quad \llbracket -est \rrbracket = \lambda \mathbf{Q}_{\langle dt, t \rangle} \lambda P_{\langle d, t \rangle} [\exists d [P(d) \wedge \forall Q \in \mathbf{Q} [Q \neq P \rightarrow \neg Q(d)]]]$$

- This operator has the right monotonicity properties. Let's prove that it is SDE.
- Proof :

Suppose  $\mathbf{Q}_1 \Rightarrow \mathbf{Q}_2$ , and take any predicate  $P$  such that  $\llbracket -est \rrbracket(\mathbf{Q}_1)(P)$  and  $\llbracket -est \rrbracket(\mathbf{Q}_2)(P)$  are well defined, in particular  $P \in \mathbf{Q}_1$  and  $P \in \mathbf{Q}_2$ .

We want to show  $\llbracket -est \rrbracket(\mathbf{Q}_2)(P) \rightarrow \llbracket -est \rrbracket(\mathbf{Q}_1)(P)$

Suppose  $(H_1): \exists d [P(d) \wedge \forall Q \in \mathbf{Q}_2 [Q \neq P \rightarrow \neg Q(d)]]$

Take such a  $d$  and take any  $Q$  in  $\mathbf{Q}_1$  (i.e.  $\mathbf{Q}_1(Q)$  is true) with  $Q \neq P$ . We want to show that we have  $\neg Q(d)$

Since  $\mathbf{Q}_1 \Rightarrow \mathbf{Q}_2$ , we have also  $Q$  in  $\mathbf{Q}_2$

Then we have  $P(d)$ ,  $Q \in \mathbf{Q}_2$  and  $Q \neq P$ . From  $(H_1)$ , we get  $\neg Q(d)$

□

- The operator is SDE. Still we should discuss what kind of situation the entailment  $\mathbf{Q}_1 \Rightarrow \mathbf{Q}_2$  represents, linguistically.

## Loccioni (2019)

- (10) a. Lenuccia is the kindest possible. (predicative)  
 b. [ MAX 3 possible <for Lenuccia to be  $t_3$  -kind >] [ 2 Lenuccia (is)  $t_2$  -kind ]
- Like Romero, Loccioni collects the set of possible degrees ([3 possible <for Lenuccia to be  $t_3$  -kind >]).
  - But for the predicative meaning, she determines the max of this set of degrees.
  - Then this maximum degree is used to saturate the degree property [ 2 Lenuccia (is)  $t_2$  -kind ], in order to express that the individual (Lenuccia) reached this maximum degree in the actual world.
  - In this proposition, it is not clear what the SUP operator is. But we can test a kind of monotonicity, as follows.
  - Given two sets of degrees,  $Q_1$  and  $Q_2$ , such that  $Q_1 \subseteq Q_2$ , we have  $\text{MAX}(Q_1) \leq \text{MAX}(Q_2)$ . Suppose that Lenuccia is  $d_2$ -kind, with  $d_2 = \text{MAX}(Q_2)$ . Then she is at least  $d_1$ -kind, with  $d_1 = \text{MAX}(Q_1)$ .

## Counterparts issue

- Romero and Loccioni (2019) only provide interpretations with the strong constraint that the individuals considered through possible worlds necessarily have a counterpart in the actual world.
- For instance, in the predicative example *Lenuccia is the kindest possible*, in Loccioni (2019), the kindness of Lenuccia in the actual world is compared to the kindness of "herself" in possible worlds.
- Romero compares John climbing a mountain in possible worlds just to John climbing a mountain in the actual world, in the attributive example *John climbed the highest possible mountain*.
- A constraint that boils down to having a counterpart is there in the so-called generic case too, where Romero implements a wide scope universal  $\forall y$  on individuals.

## Lost of information

- By introducing sets of degrees (type  $\langle d, t \rangle$ ), or sets of degree properties (type  $\langle dt, t \rangle$ ), Loccioni (2019) and Romero lose information on the relation between individuals, worlds and amounts.
- These shortcomings in the way information is collected and used are arguably due to the ACD approach to modal superlative clauses, broadly adopted since Larson (2000).
- One question for syntacticians is whether an alternative syntactic analysis can be found.

## Tovena and Fleury (2024)

## $SUP_{mod}$ on equivalence relation of world-individual pairs

- Tovena and Fleury's analysis (2023, 2024) relies on a comparison class that is an equivalence relation of world-individual pairs sorted by the amount they are associated with.
- The class is built using a function  $F$  that represents necessary and sufficient information to carry out the comparison, see (11a), and is distinct from the modal superlative operator  $SUP$  responsible for the comparison itself, see (11b).

$$(11) \quad a. \quad F : \lambda c' \lambda q' [c' = \{(w', x') : w' \in Acc(w) \wedge Q(w', x', q')\}]$$

b.  $SUP_{mod}$  (to be revised) :

$$\lambda F \lambda x [\exists q [\exists c [(w, x) \in c \wedge F(c)(q)] \wedge \forall c' [(\exists q' [F(c')(q')]) \wedge c \neq c' \rightarrow q' < q]]]]$$

- $F$  associates an amount with a set of world-individual pairs grouped into equivalence classes and can work as a comparison class.  $Q$  in (11a) expresses the restriction verified for any individual in any world, without hierarchising the two.
- Grouping world-individual pairs by amount in  $c'$  reflects the equative reading.  $SUP$  expresses the facts that for a given function  $F$  and entity  $x$ , there exists one amount  $q$  associated with  $x$ , and that amounts associated with the other  $x'$  are smaller.



## SUP<sub>mod</sub> revised

- By construction,  $F$  keeps information on the relation between individuals  $x'$ , groups  $c'$  of world-individual pairs and amounts  $q'$ . This makes possible the abstraction  $\lambda x$ , which is not available in  $\text{est}^{\text{H2}}$  style.
- Before we test the monotonicity properties of SUP<sub>mod</sub>, (11b) is revised as in (12a). The subformula that makes it possible to associate the amount  $q$  with the equivalence class of pairs of individual  $x$  in evaluation world  $w$ , is a presupposition of existence of  $q$  (12b), and has to be taken out, in line with von Stechow (1999).

$$(12) \quad \begin{aligned} \text{a. } & \text{SUP}_{\text{mod}} \text{ (revised)} : \\ & \lambda F \lambda x [\forall q, q', c' [(\exists c [(w, x) \in c \wedge F(c)(q) \wedge F(c')(q') \wedge c \neq c']) \rightarrow q' < q]] \\ \text{b. } & \text{Presupposition} : \exists q [\exists c [(w, x) \in c \wedge F(c)(q)]] \end{aligned}$$

- The existential on  $c$  and the part  $F(c)(q) \wedge F(c')(q')$  in the formula have no other purpose than to make the bridge between the individual  $x$  in the actual world  $w$  and its equivalence class  $c$ .
- The use of  $c \neq c'$  with  $q' < q$  instead of a simple  $q' \leq q$  is not crucial, since we deal with equivalence classes by amount.
- Apart from the two previous remarks, (12a) is a standard way to get the Sup element among the "comparison class"  $F$ .

## SUP<sub>mod</sub> revised

- We could benefit from splitting the operator into two parts, a  $\text{SUP}'_{\text{mod}}$  operator at the level of equivalence classes in (13) and an operator at the level of individuals in (14), as we proposed at RALFe 2025 conference, with the aim of unifying ordinary and modal superlative operators.

$$(13) \quad \text{SUP}'_{\text{mod}}: \lambda F \lambda c [\forall q, q', c' [(F(c)(q) \wedge F(c')(q')) \rightarrow q' \leq q]]$$

$$(14) \quad \lambda G \lambda x [(w, x) \in \cup \{c : G(c)\}]$$

- The operator  $\text{SUP}'_{\text{mod}}$  in (13) applies to  $F$  and gives a set of equivalence classes  $c$ , actually restricted to the class associated with the highest degree.
- The operator in (14) applies to  $\text{SUP}'_{\text{mod}}(F)$  and says that the pair  $(w, x)$ , that is the individual  $x$  in the world of evaluation  $w$ , belongs to the union of the equivalence classes, actually to the unique equivalence class associated with the highest degree.
- The operator in (14) possibly plays the role of a kind of determinant.
- The presupposition for (13) may be adapted as  $\exists q[F(c)(q)]$ .

# $SUP_{mod}$ is SDE

$$(11a) \quad F : \lambda c' \lambda q' [c' = \{(w', x') : w' \in Acc(w) \wedge Q(w', x', q')\}]$$

$$(12a) \quad SUP_{mod} \text{ (revised)} : \\ \lambda F \lambda x [\forall q, q', c' [(\exists c [(w, x) \in c \wedge F(c)(q) \wedge F(c')(q') \wedge c \neq c']) \rightarrow q' < q]]$$

- To prove that the operator  $SUP_{mod}$  is Strawson downward-entailing (SDE), assume the Strawson entailment  $F_1 \Rightarrow F_2$ .
- The antecedent of the implication in (12a) expressed for  $F_1$  implies the antecedent expressed for  $F_2$ , which is the key to demonstrating  $SUP_{mod}(F_2) \Rightarrow SUP_{mod}(F_1)$ .
- Proof :

Let's take any  $x$  and consider that we have  $SUP_{mod}(F_2)(x)$ , that is:

$$(H_1) : \forall q, q', c' [(\exists c [(w, x) \in c \wedge F_2(c)(q) \wedge F_2(c')(q') \wedge c \neq c']) \rightarrow q' < q]$$

We want to show that we have  $SUP_{mod}(F_1)(x)$ . Let's take any  $q, q', c'$  and consider that we have  $\exists c [(w, x) \in c \wedge F_1(c)(q) \wedge F_1(c')(q') \wedge c \neq c']$ , and let's show that we have  $q' < q$

As we have  $F_1 \Rightarrow F_2$ , we have also  $\exists c [(w, x) \in c \wedge F_2(c)(q) \wedge F_2(c')(q') \wedge c \neq c']$ , and by hypothesis (H1), we get  $q' < q$

□

## $SUP_{mod}$ is not SUE

$$(11a) \quad F : \lambda c' \lambda q' [c' = \{(w', x') : w' \in Acc(w) \wedge Q(w', x', q')\}]$$

$$(12a) \quad SUP_{mod} \text{ (revised)} : \\ \lambda F \lambda x [\forall q, q', c' [(\exists c [(w, x) \in c \wedge F(c)(q) \wedge F(c')(q') \wedge c \neq c']) \rightarrow q' < q]]$$

- $SUP_{mod}$  is not SUE

- Proof :

Suppose  $F_1 \Rightarrow F_2$ , with  $F_1 \neq F_2$ . And suppose there exists a class  $c^*$  and an amount  $q^*$  s.t.  $F_2(c^*, q^*)$ , and there exists no  $c'$  satisfying  $F_1(c', q^*)$ .

Take the particular case where  $q^*$  is greater than all the amounts  $q'$  reached by  $F_1$ .

Let entity  $x$  be s.t.  $SUP_{mod}(F_1)(x)$ . Such  $x$  reaches a certain amount  $q'$  which is strictly less than  $q^*$ . Hence, we cannot have  $SUP_{mod}(F_2)(x)$ , and this shows that  $SUP_{mod}$  is not SUE.

□

## Level of abstraction

$$(11a) \quad F : \lambda c' \lambda q' [c' = \{(w', x') : w' \in Acc(w) \wedge Q(w', x', q')\}]$$

$$(12a) \quad \text{SUP}_{mod} \text{ (revised)} : \\ \lambda F \lambda x [\forall q, q', c' [(\exists c [(w, x) \in c \wedge F(c)(q) \wedge F(c')(q') \wedge c \neq c']) \rightarrow q' < q]]$$

- The proof presented above applies Strawson entailments to the function  $F$  from equivalence classes to amounts, i.e. a rather abstract function.
- The question arises as to the degree of abstraction at which the monotonicity properties must be verified.
- We note that it feels difficult to come up with linguistic examples of functions  $F_1$  and  $F_2$  as defined in (11a) that are in a Strawson entailment relation  $F_1 \Rightarrow F_2$ .
- Technically speaking, this is because this relation involves equivalence classes, and whenever the accessibility function  $Acc$  or the predicate  $Q$  are altered for the purpose of monotonicity checking, the content of the equivalence class associated with a given amount  $q$  is affected.

## Entailment at the level of equivalence classes

- However, an entailment relation can still be verified at the level of the world-individual pairs  $(w', x')$  which themselves remain directly accessible using the function  $F$ , rather than at the level of the equivalence classes  $c'$  that contain them.
- The world-individual pairs  $(w', x')$  that we need to consider are the elements of the equivalence classes in the first argument of  $F$ , and the amounts associated to these world-individual pairs are the amounts associated to the equivalence classes by the function  $F$ .
- There is no loss of information and no need to make a copy of any content of the sentence to retrieve this information, as per ACD. The entailment can be expressed as follows: if  $(w', x')$  belongs to class  $c_1$  associated with amount  $q$  via function  $F_1$ , then  $(w', x')$  belongs to some class  $c_2$  associated with amount  $q$  via function  $F_2$ .
- However, it is not required that all pairs  $(w'', x'')$  in  $c_2$  belongs to  $c_1$ . At the more abstract level of equivalence classes, this entailment relation can be expressed as follows: if  $F_1(c_1)(q)$ , then there exists  $c_2$  s.t.  $c_1 \subseteq c_2$  and  $F_2(c_2)(q)$ , noted  $F_1 \dot{\Rightarrow} F_2$ .
- We will show that this relation between  $F_1$  and  $F_2$  implies the Strawson relation  $\text{SUP}_{\text{mod}}(F_2) \Rightarrow \text{SUP}_{\text{mod}}(F_1)$ . In this sense,  $\text{SUP}_{\text{mod}}$  verifies the SDE property at the level of world-individual pairs, while it verifies a quasi-SDE property only at the more abstract level of equivalence classes.

# Monotonicity (revised)

- Proof :

Suppose  $F_1 \dot{\Rightarrow} F_2$ , that is:  $F_1(c_1)(q) \rightarrow \exists c_2 [c_1 \subseteq c_2 \wedge F_2(c_2)(q)]$ , for all  $c_1, q$

Let's take  $x$  such that  $\text{SUP}_{\text{mod}}(F_2)(x)$ , that is:

(H1) :  $\forall q, q', c' [(\exists c [(w, x) \in c \wedge F_2(c)(q) \wedge F_2(c')(q') \wedge c \neq c']) \rightarrow q' < q]$

We want to show that we have  $\text{SUP}_{\text{mod}}(F_1)(x)$ . Let's take any  $q, q', c'_1$  and consider that we have  $\exists c_1 [(w, x) \in c_1 \wedge F_1(c_1)(q) \wedge F_1(c'_1)(q') \wedge c_1 \neq c'_1]$ , and let's show that we have  $q' < q$

As we have  $F_1 \dot{\Rightarrow} F_2$ , there exists  $c_2$  and  $c'_2$  such that  $c_1 \subseteq c_2$  and  $c'_1 \subseteq c'_2$  and  $F_2(c_2)(q)$  and  $F_2(c'_2)(q')$ . We have also  $(w, x) \in c_1 \subseteq c_2$

Moreover, since we have  $c_1 \neq c'_1$ , we have necessarily  $q \neq q'$ , because  $c_1$  and  $c'_1$  are equivalence classes by amount. Then we have necessarily  $c_2 \neq c'_2$  for the same reason.

Then we have  $\exists c_2 [(w, x) \in c_2 \wedge F_2(c_2)(q) \wedge F_2(c'_2)(q') \wedge c_2 \neq c'_2]$ , and by hypothesis (H1), we get  $q' < q$

□

## Discussion



## The use of equivalence classes

- Loccioni (2018) applies the -ER operator (*più*) to get and order equivalence classes of individuals with the same degree, when dealing with absolute superlatives.
- When dealing with modal superlatives, *più* orders just degrees, because the information on their association with individuals has been lost, and with that is lost the notion of equivalence class.
- Equivalence classes are also the strategy adopted by Tovenà and Fleury (2024), but they are built by grouping world-individual pairs, and are used for the semantics of the modal superlative operator. The result preserves the level of information needed to get the right truth conditions without using ACD.
- The monotonicity properties of the operator are not tackled, but one can get the right properties by taking out the presupposition, as we proposed in (12b).

## Further research

- The discussion can be pursued in at least two directions, as we might want i) to vary the entailments by plugging in tests that take up Zwarts (1998) hierarchy, or ii) to look at empirical facets of the NPIs licensing power.
- The (modal) superlative operators discussed all have the right monotonicity properties, but they do not seem to license NPIs in all languages (Corblin and Tovenà 2003, Hoeksema 2012, Loccioni 2018 i.a.), and evidence of NPIs licensed by modal superlatives is scanty.

## Example

- We conclude by discussing the (rare) example [(35i)] from Bumford and Sharvit, given in (15), where the modal superlative interpretation seems available with *any*, despite *possible* being prenominal and doubts concerning whether *any* is NPI or free choice.
 

(15) Our goal with this satellite is to capture the best possible image of any asteroid in the Kuiper Belt
- In (15), an accessible world in which a carbonaceous asteroid image of quality  $q$  is captured is a fortiori an accessible world in which an asteroid image of such quality  $q$  is captured. If the best asteroid image is captured, and if this is a carbonaceous asteroid image, then we can say that the best carbonaceous asteroid image is captured.
- If *any* is actually an NPI, then the  $SUP_{mod}$  operator seems to have the SDE property, at the level of world-individual pairs.

## Example

- Note that the modification expressed by *any* in this example concerns the predicate *asteroid*, which is part of the restriction on the type of situation expressed by the predicate  $Q$  and constitutes a restriction on the individuals in the accessible worlds (Tovena and Fleury, 2024).
- The use of an NPI for the modality, although seemingly difficult, should not be completely ruled out.
- A study of a corpus of modal superlatives will make it possible to determine the interest of quasi-SDE property, and to verify which phenomenon this property could reflect in the language.

## References

- Bumford, D. and Sharvit, Y. (2022). Negative polarity items in definite superlatives. *Linguistic Inquiry*, 53(2):255–293.
- Corblin, F. and Toven, L. (2003). L'expression de la négation dans les langues romanes. In Godard, D., editor, *Les langues Romanes: problèmes de la phrase simple*, pages 281–343. CNRS Editions, Paris.
- von Fintel, K. (1999). NPI-licensing, Strawson-entailment, and context-dependency. *Journal of Semantics*, 16(2):97–148.
- Heim, I. (1999). Notes on superlatives. unpublished ms. MIT.
- Hoeksema, J. (2012). On the natural history of negative polarity items. *Linguistic Analysis*, 38:3–33.
- Larson, R. (2000). ACD in AP. In *Proceedings of the 19th Annual West Coast Conference on Formal Linguistics (WCCFL 19)*, pages 1–15.
- Loccioni, N. (2018). *Getting 'the most' out of Romance*. PhD thesis, University of California Los Angeles USA.
- Loccioni, N. (2019). The Romance of modal superlatives as degree descriptions. In *Proceedings of Semantics and Linguistic Theory (SALT 29)*, pages 219–237.
- Romero, M. (2013). Modal superlatives: a compositional analysis. *Natural Language Semantics*, 21(1):79–110.

## References

- Tovena, L. M. and Fleury, D. (2023). Situations and modality in predicative modal superlatives. *Working papers in linguistics and oriental studies (QULSO)*, 9:195–211.
- Tovena, L. M. and Fleury, D. (2024). The anchor of a modal superlative and the individual vs stage level reading of the adjective. *Isogloss*, 10:1–29.
- Zwarts, F. (1998). Three types of polarity. In Hamm, F. and Hinrichs, E., editors, *Plurality and quantification*, pages 177–238. Kluwer, Dordrecht.