Constraining Meaning – Category-Theoretic Perspectives on Semantics

Miriam S. Schmidt
University of Göttingen
miriamsophie.schmidt@uni-goettingen.de

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Objectives

Perspective on (hyper)intensionality

Motivate the idea that (hyper)intensional phenomena clash with the extensionality of functions operating on sets

A semantic category and constraints on it

Use the Yoneda Lemma for understanding meaning through relations and make computations flexibly context-sensitive via constraints

Framework comparison

Use the Yoneda Lemma for comparing a CT-approach with classical PWS

Overview/ Work in progress!

Overview

1. Knowing *that*, but not *why*
2. Semantics without sets
   - Key idea – The Yoneda perspective
   - A semantic category
3. Constraining meaning
   - Extensional vs. intensional
   - Hyperintensionality
   - Subject-matter
   - Factivity
   - DP entailment
4. More on Yoneda

Possible Worlds

Standard PWS analysis (Kratzer 2006 & 2022, Hegarty 2011, Djärv 2023, . . .) of verbs embedding a content p (*believe*, *know*, or *regret* etc.), based on Hintikka (1969), (e.g., $\text{ATT}{=}\text{DOX}$ for *believe*):

(1) $[\![\textit{attitude}]\!] = \lambda p \lambda x \lambda w. \forall w' : w' \in \text{ATT}_{x,w} \rightarrow w' \in p$

(2) a. Yoda believes Anakin is a Sith.

b. $\lambda w. \forall w' : w' \in \text{DOX}_{\text{Y},w} \rightarrow w' \in \{w \mid \text{Anakin is a Sith in } w\}$

(Im)Possible worlds

Individuation of possible world sets leads to problems in a hyperintensional setting:

(3) Yoda is a logician and knows that p and $p \wedge (q \to p)$ are logically equivalent. Ahsoka, young and stubborn, also believes p, but thinks that $p \wedge (q \to p)$ means something different and is skeptic about it.

a. Yoda believes that p and $p \wedge (q \to p)$.

b. Ahsoka believes that p and $p \wedge (q \to p)$

- ↯ Worlds for p and $p \wedge (q \to p)$ are identical, but subject-matter (c.f. Yablo 2014) is not.
- ↯ Related: *bachelor* / *unmarried male*, *buy*(x, y, z)/*sell*(z, y, x), ...

(Im)Possible Worlds

(4) Theseus' starship flies through the galaxy and exchanges all of its parts during travel due to repair work. Watto, a junk dealer, quickly detects all repairs and does not recognize the identity of the ship that just arrived on Tatooine. The Jedi Obi-Wan is simply happy about the return of Theseus and his crew. He recognizes the identity of the ship.

  a. Watto correctly believes that Theseus' starship did not arrive.

  b. Obi-Wan correctly believes that Theseus' starship arrived.

(5) Anakin once was a famous Jedi and general in the Clone Wars. Then he turned to the dark side and, using dark forces, grew even more powerful. For Rey, being a hero is to be powerful and fierce. For Kylo, being a hero is to be courageous and kind-hearted.

  a. Rey believes that Anakin was a hero.

  b. Kylo believes that Anakin was a hero.

- $\lightning$ Again: identical worlds; here due to different concepts
- $\lightning$ possibly multiple concepts simultaneously available

Change your type or your world

Previous solutions for (hyper-)intensionality effects:

1. Change definition of possible world/proposition (Cresswell 1985) and/or notion of truth (Fine 2017, Moltmann 2024)
2. Change type (system)
   - (context-)dependent types (Cooper 2023)
   - less types, but sorts (Chierchia/Turner 1988, Liefke/Werning 2018)
   - (monadically) 'lift' types (≈ add variables for perspective) (Asudeh/Giorgolo 2020, Elliott 2023)

- Essentially: **change the input** of functions

Knowing *that*, but not *why*

What is difficult in analysis, isn't difficult for humans:

- We know that it's Ahsoka (i.e., not a logician) juding the propositions, so she might judge them differently.
- We know Obi-Wan and Watto have different criteria for the identity of starships.
- We know Kylo and Rey have different concepts of what it means to be hero.

$\Rightarrow$ We don't forget the context (/previously evaluated arguments) while computing meaning.

Can we incorporate that context into our computations without transfering the complexity into the input?

Functions and extensionality

A first answer: Not really.

- Most standard approaches operate on some sort of set.
- Functions on these sets are extensional in nature:

$$f(2+2) = f(4) = f(6-2) = f(2 \cdot \sum_{k=0}^{\infty} \frac{1}{2^k}) = \dots$$

- The function 'forgets' the steps of computation.
- Not a problem for extensional contexts:

(6) CONTEXT: Luke is Leia's brother.
Leia kissed Luke. ⇔ Leia kissed her brother.

- But for intensional contexts (and the phenomena it causes) this extensionality of functions in set theory can't reflect the intensional side effects (hence the modified input).

Semantics without sets

- But we need functions – can we have functions without sets?

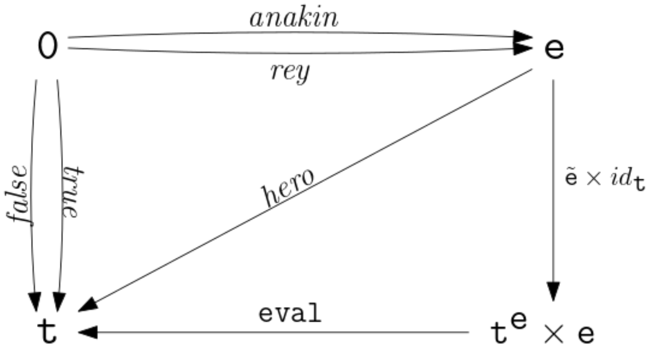
$\Rightarrow$ Function-like objects on set-like-objects

$\Rightarrow$ Category Theory (CT) offers an alternative perspective (not just a tool!) for modelling semantics and especially intensionality (Peruzzi 2006)

- CT as a "meta mathematics": More general than conceptualisations based on Set Theory (ST), no sets, just 'collections'.
- In CT, equivalence is not defined point-wise (no $x = y$ or $M = M'$ if $\forall x : x \in M \leftrightarrow x \in M'$), objects remain 'anonymous'.
- The perspective is fully relational: Equality (up to isomorphism) is defined via morphisms between objects (or functors between categories).
- Objects are then defined via the totality of their environment:
  $\Rightarrow$ **Yoneda Lemma**

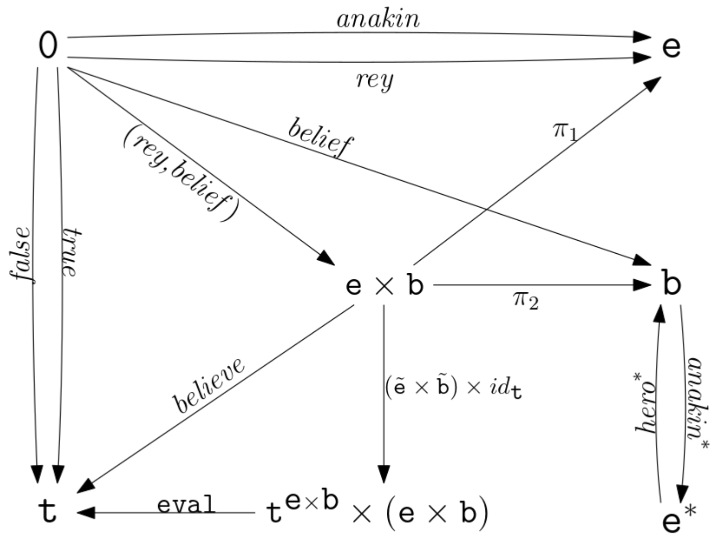
Semantics without sets

As Asudeh/Giorgolo (2020) have sketched, we can do compositional semantics in CT just like in ST:



Now let's include (belief) content!

A semantic category



Constraining meaning

- Everything is based on objects and morphisms – 'regular' objects (i.e., types) and the computations on them (function objects and `eval`). We can thus rationalize about objects **and** computations on them on the same level.
- No absolute stance on types: `b` could also be assumed to be `e` or `p`. (I am following Moltmann's ideas here and implicitly assume *believing* brings about a `b`*elief*, an attitudinal object.)
- What types are needed? A *skeletal category* collapses isomorphic objects (empirical data!).
- For now: only function application.
  Semantics beyond can be done via constraints on morphisms:

Extensional vs. intensional

How to discern extensional objects (e.g., *punch*) from intensional ones (e.g. *believe*)?
Testing for robust reference:

(7) a. Yoda punches Anakain.

b. The storm trooper punches Anakin.
$\models$ Yoda and the storm trooper punch the same thing.

(8) CONTEXT: Yoda already knows that Anakin is Darth Vader, the storm trooper doesn't.

a. Yoda believes that Anakin is a Sith.

b. The storm trooper believes that Anakin is a Sith.
$\not\models$ Yoda and the storm trooper believe the same thing.

Extensional vs. intensional

Let a_1, a_2 be two entities, x an object towards a_1, a_2 stand in a relation to. Then:

$$(a_1, x) \neq (a_2, x) \Rightarrow \pi_2 \circ (a_1, x) = \pi_2 \circ (a_2, x) \quad \text{extensional case}$$
$$(a_1, x) \neq (a_2, x) \not\Rightarrow \pi_2 \circ (a_1, x) = \pi_2 \circ (a_2, x) \quad \text{intensional case}$$

Most of the times, the intensional case can be modelled with *monic* morphisms:

$$(a_1, x) \neq (a_2, x) \Rightarrow \pi_2 \circ (a_1, x) \neq \pi_2 \circ (a_2, x)$$

Hyperintensionality

How to account for hyperintensionality induced by different concepts?
We know when to expect hyperintensionality:
In contexts of intensional objects (in out category: b).

$$c^* \circ \textit{belief} = c^* \circ \pi_2 \circ (\mathsf{e}, \mathsf{b}) \Rightarrow c^* = c_{\mathsf{e}}$$

- What is c?

⇒ The content, i.e. (composite) morphisms, describing the intensional object (e.g., *Anakin is a hero* for *believe*, *a starship* for *seek*, . . .)

- What is c_{e}?

⇒ The *monadic* interpretation of the content c (in that case: the `reader` monad, adding a perspectival index), c.f. Asudeh/Giorgolo (2020).

Subject-matter

Subject-matter (possible QUDs) can be 'collected' by tracing the (non-computational) morphisms.

(9) Anakin is a hero.
SUBJ-MTR: $\{$*hero*,*anakin*,*hero* $\circ$ *anakin*$\}$

(10) (Ahsoka believes that) p and $p \wedge (q \rightarrow p)$.
SUBJ-MTR: $\{p, q, \rightarrow \circ(q, p), \wedge \circ (p, \rightarrow \circ(q, p))\}$

- Could be modeled in semantics directly (`writer`-monad) or updated in an interface to (syntax and) pragmatics (opinions?).

Factivity

- Factive predicates induce a necessary relation of content with truth.
- Deriving a constraint on factivity of a predicate f:

$$f \circ (\mathtt{e}, c) = \mathtt{eval} \circ h \circ (\mathtt{e}, c)$$
$$\Rightarrow \exists c' \in Hom(\mathtt{0}, \mathtt{t}) : \eta(c') = \pi_2 \circ (\mathtt{e}, c) \ \& \ c' = \textit{true}$$

$$(h = (\tilde{\mathtt{e}} \times \tilde{\mathtt{b}}) \times id_{\mathtt{t}},$$
$$\eta = \text{unit for } \mathtt{reader} \text{ monad})$$

- Any factive evaluated predicate, e.g., *know* or *regret* that applies to a tuple of a logical subject e and a content c, implies that c commutes with the morphism *true* – regardless of whether f itself is true for e and c.

DP entailment

(11) a. $\models$ b.

a. Yoda *attitude* {the story,the claim,...} that p.

b. Yoda *attitude* that p.

- As has widely been noted, (11) holds for *believe*-type verbs, but not for *know*.
- CT has a direct answer to the problem: It 'collapses' objects, that are isomorphic to each other, i.e., if there is an isomorphism between them. If there is no such morphism, we need to distinguish them (c.f. Mazur 2008).

DP entailment

(Non-)Isomorphic relations between *knowledge*/*belief* and Content DPs:

(12) a. Yoda believes the story that p.
b. There is a story that p and Yoda believes that p.

(13) a. Yoda knows the story that p.
b. There is a story that p and Yoda knows that p.

- Both a.$\models$b. and b.$\models$a. holds for *believe*, but not for *know*.
- *believe*-type verbs are transparent w.r.t. content, so the objects of *belief* and *story* collapse.
- We don't even need a constraint! The distinction follows due to the availability/lack of an isomorphism.

More on Yoneda

Yoneda Lemma

For all functors $F : \mathcal{C} \to \mathcal{S}et$ and every object c in $\mathcal{C}$, there is a natural bijection (natural in F und c):[1]

$$Nat(\mathcal{C}(c, -), F) \approx Fc$$

Assume c to be an object in our category of truth and Fc to be the set of worlds corresponding to that c:

Category theoretical description (left) and set-based description (right) can be 'translated' into each other. They are isomorphic.

[1] Cited after Riehl (2016).

Conclusion

Understanding meaning in category theory

- allows for constraints on computations directly rather than on inputs only.
- brings back the *why* into our analysis.
- renders the descriptions of (hyper-)intensional problems flat and easy.
- enables us to compare frameworks.

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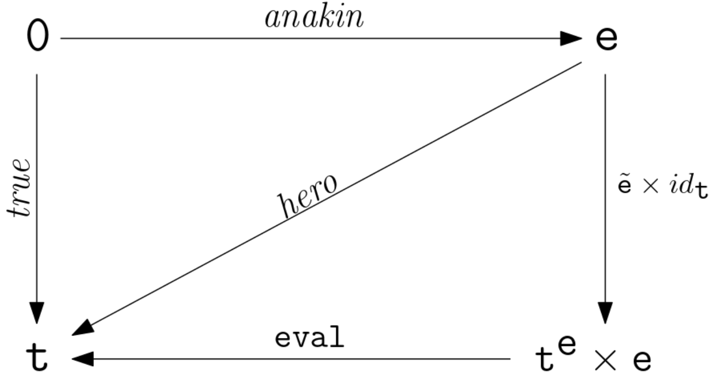
System of the semantics

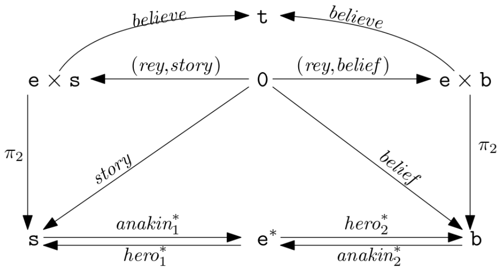
- The meaning of a sentence is given through the diagram and the way, in which it commutes.
- All morphisms must and do compose.
- A (framework) ontology states existence and properties of objects and morphisms. (HPSG: *Signature*)
- All constraints must be met, otherwise undefined. (HPSG: *Theory*)
  - Global constraints (extensional/intensional,. . .)
  - Lexical constraints (*buy* $\Leftrightarrow$ *sell*)

Notion of truth

A predicate *hero* applied to *anakin* is true, iff the diagram commutes:



DP entailment



(numbering only for readability)

$$hero_1^* \circ anakin_2^* \circ hero_2^* \circ anakin_1^* = id_{s}(= \pi_2 \circ (rey, story))$$

- "The story that Anakin is a hero, for which it is the case that Rey believes that Anakin is a hero, is Rey's believed story."

$$hero_2^* \circ anakin_1^* \circ hero_1^* \circ anakin_2^* = id_{b}(= \pi_2 \circ (rey, belief))$$

- "The belief that Anakin is a hero, for which it is the case that Rey believes the story that Anakin is a hero, is Rey's belief."

Concepts in *belief* and *story* must match → same embedded ontology e^*.

The category of meaning

Properties of the category:

- cartesian closed (terminal object, (finite) products and exponentials exist for all types)
- small (/finite)
- based on Curry-Howard-Lambek correspondence: objects are types, morphisms operate on them
- 'built-in' currying of functions (c.f. universal construction for defining exponentials!)