A uniform semantics for connexive and paraconsistent Nelson logics

Umberto Rivieccio

Departamento de Lógica, Historia y Filosofía de la Ciencia
Universidad Nacional de Educación a Distancia
Madrid (Spain)

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  - Aristotle's thesis $\neg(\neg A \to A)$

- Connexive logics are often paraconsistent ($A, \neg A \nvdash B$), or even contradictory, i.e. may admit a formula A such that $\vdash A$ and $\vdash \neg A$.

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- pN and C essentially differ only regarding negated conditionals:

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- Could we draw a more precise formal comparison?

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  Our algebraic analysis will suggest that this is not the case.

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- Both twist constructions essentially coincide except for the representation of the implication operator (see below).

Algebraic semantics

The twist-algebra construction

Given an implicative lattice $\mathbf{L} = \langle L; \wedge, \vee, \to 1 \rangle$, the full twist-algebra over $\mathbf{L}$ is the algebra

$$\mathbf{L}^{\bowtie} = \langle L \times L; \wedge, \vee, \to_{N/C}, \neg \rangle$$

with operations given by:

$$\begin{aligned}
\langle a_1, a_2 \rangle \wedge \langle b_1, b_2 \rangle &:= \langle a_1 \wedge b_1,\ a_2 \vee b_2 \rangle \\
\langle a_1, a_2 \rangle \vee \langle b_1, b_2 \rangle &:= \langle a_1 \vee b_1,\ a_2 \wedge b_2 \rangle \\
\langle a_1, a_2 \rangle \to_{\mathrm{pN}} \langle b_1, b_2 \rangle &:= \langle a_1 \to b_1,\ a_1 \wedge b_2 \rangle \\
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N4-lattices arise as algebras of type $\langle A, \wedge, \vee, \to_N, \neg \rangle$,
and C-algebras are those of type $\langle A, \wedge, \vee, \to_C, \neg \rangle$.

Comparing N4-lattices and C-algebras

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- In general, neither an N4-lattice need have a term-definable C-algebra structure, nor the other way around.
- In particular, the two classes of algebras (hence the two logics) are not definitionally equivalent.
- However, it is not hard to view both constructions as two instances of a common one...

Abstracting N4-lattices and C-algebras

The idea

Define twist-algebras similarly as before for the language $\{\wedge, \vee, \neg\}$, but let

$$\langle a_1, a_2 \rangle \to \langle b_1, b_2 \rangle := \langle a_1 \to b_1, a_1 \ominus b_2 \rangle$$

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Abstract properties

1. $x = 1 \ominus x$.
2. $(x \wedge y) \ominus z = x \ominus (y \ominus z)$.
3. $x \leq y$ entails $z \ominus x \leq z \ominus y$.
4. $(x \vee y) \ominus z \leq (x \ominus z) \vee (y \ominus z)$.
5. $(x \leftrightarrow y) \ominus x \leq (x \leftrightarrow y) \ominus y$.
6. $x \to y \leq (z \ominus x) \to (z \ominus y)$.
7. $x \leftrightarrow y \leq (x \ominus z) \to (y \ominus z)$.

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- A twist construction/representation can also be developed if we drop involutivity (double negation law).

An application

The algebra of ordinary discourse

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$\wedge_{\mathrm{OL}}$	$1/2$	1	0
$1/2$	$1/2$	1	0
1	1	1	0
0	0	0	0

$\to_{\mathrm{OL}}$	$1/2$	1	0
$1/2$	$1/2$	1	0
1	$1/2$	1	0
0	$1/2$	$1/2$	$1/2$

	$\neg_{\mathrm{OL}}$
$1/2$	$1/2$
1	0
0	1

[The disjunction is defined by $x \vee_{\mathrm{OL}} y := \neg_{\mathrm{OL}}(\neg_{\mathrm{OL}} x \wedge_{\mathrm{OL}} \neg_{\mathrm{OL}} y)$].

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which give us:

$$\neg_{\mathrm{OL}} \langle a_1, a_2 \rangle := \langle a_2,\, a_1 \rangle$$
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- Investigate the relationship between the present framework and Logic(s) of Ordinary Discourse.

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