N∅thing is Logical

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N∅thing is logical (Nihil)

- Goal of the project: a formal account of a class of natural language inferences which deviate from classical logic
- Common assumption: these deviations are not logical mistakes, but consequence of pragmatic enrichments (Grice)
- Strategy: develop *logics of conversation* which model next to literal meanings also pragmatic factors and the additional inferences which arise from their interaction
- Novel hypothesis: **neglect-zero** tendency (a cognitive bias rather than a conversational principle) as crucial factor
- Main conclusion: deviations from classical logic consequence of enrichments albeit not (always) of the canonical Gricean kind

Non-classical inferences

Free choice (FC)

(1) FC: $\Diamond(\alpha \vee \beta) \rightsquigarrow \Diamond\alpha \wedge \Diamond\beta$

(2) Deontic FC inference [Kamp 1973]

a. You may go to the beach *or* to the cinema.
b. $\rightsquigarrow$ You may go to the beach *and* you may go to the cinema.

(3) Epistemic FC inference [Zimmermann 2000]

a. Mr. X might be in Victoria *or* in Brixton.
b. $\rightsquigarrow$ Mr. X might be in Victoria *and* he might be in Brixton.

Ignorance

(4) The prize is either in the garden *or* in the attic $\rightsquigarrow$ speaker doesn't know where [Grice 1989, p.45]

(5) ? I have two *or* three children.

- In the standard approach, ignorance inferences are conversational implicatures
- Less consensus on FC inferences analysed as conversational implicatures; grammatical (scalar) implicatures; semantic entailments; . . .

Note: Adding FC to classical modal logic implies the equivalence of any two possibility claims:

$$\Diamond a \Rightarrow_{CML} \Diamond(a \vee b) \Rightarrow_{FC} \Diamond b$$

Novel hypothesis: neglect-zero

- FC and ignorance inferences are [$\neq$ semantic entailments]
  - Not the result of Gricean reasoning [$\neq$ conversational implicatures]
  - Not the effect of applications of covert grammatical operators [$\neq$ grammatical (scalar) implicatures]
- But rather a consequence of something else speakers do in conversation, namely,

  NEGLECT-ZERO
  when interpreting a sentence speakers construct models depicting reality[1] and in this process tend to neglect models that verify the sentence by virtue of an empty configuration (*zero-models*)

- Tendency to neglect zero-models follows from the difficulty of the cognitive operation of evaluating truths with respect to empty witness sets [Nieder 2016; Bott *et al*, 2019[2]]

[1] Johnson-Laird (1983) *Mental Models*. Cambridge University Press.
[2] Bott, O., Schlotterbeck, F. & Klein U. 2019. Empty-set effects in quantifier interpretation. *Journal of Semantics*, 36, 99–163.

Novel hypothesis: neglect-zero

Illustration

(6) Every square is black.

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [△, △, △]; [▲, ▲, ▲]; . . .

Zero-models in (6-c) verify the sentence by virtue of an empty set of squares

(7) Less than three squares are black.

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [□, □, □]; [■, ■, ■]; [△, △, △]; [▲, ▲, ▲]; . . .

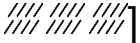
Zero-models in (7-c) verify the sentence by virtue of an empty set of black squares

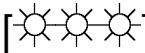
- Cognitive difficulty of zero-models confirmed by experimental findings and connected to / can be argued to explain:
  - the special status of 0 among the natural numbers [Nieder, 2016]
  - why downward-monotonic quantifiers are more costly to process than upward-monotonic ones (*less* vs *more*) [Bott, Schlotterbeck *et al*, 2019]
  - Existential Import (every A is B ⇒ some A is B) & Aristotle's Thesis (NEVER: if not A, then A)
- Core idea: FC, ignorance and other enriched interpretations prominently explained by neo-Gricean or grammatical tools, are instead a consequence of a neglect-zero tendency

Novel hypothesis: neglect-zero effects on disjunction

Illustrations

(8) It is raining.

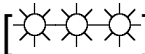
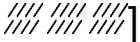
a. Verifier: []

b. Falsifier: []

c. Zero-models: none

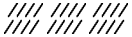
(9) It is snowing.

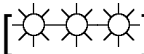
a. Verifier: []

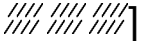
b. Falsifier: []; []; . . .

c. Zero-models: none

(10) It is raining or snowing.

a. Verifier: [ | ] [⇐ “split” state]

b. Falsifier: []

c. **Zero-models**: []; []

- Two zero-models in (10-c): verify the sentence by virtue of an empty witness for one of the disjuncts;
- Neglect-zero hypothesis: ignorance and FC effects arise because such zero-models, where only one of the disjuncts is depicted, are cognitively taxing and therefore kept out of consideration;
- Split state in (10-a): simultaneously entertains different (possibly conflicting) alternatives.

A new conjecture: no-split

A closer look at the disjunctive case

(11) It is raining or snowing.

a. Verifier: [//// //// //// | ❄❄❄] [$\Leftarrow$ "split" state]

b. Falsifier: [☼☼☼]

c. **Zero-models**: [//// //// ////]; [❄❄❄]

- Split states: multiple alternatives processed in a parallel fashion $\mapsto$ also a cognitively taxing operation (increased working memory load)

  No-split conjecture [Klochowicz, Sbardolini & MA, SuB 2025]
  the ability to split states (entertain multiple alternatives) is developed late

- Combination of neglect-zero + no-split can explain non-classical inferences observed in pre-school children [Singh *et al* 2016; Cochard 2023; Bleotu *et al* 2024]

  (12) The boy is holding an apple or a banana = The boy is holding an apple and a banana $(\alpha \vee \beta) \equiv (\alpha \wedge \beta)$

  (13) The boy is not holding an apple or a banana = The boy is neither holding an apple nor a banana $\neg(\alpha \vee \beta) \equiv \neg\alpha \wedge \neg\beta$

  (14) Liz can buy a croissant or a donut = Liz can buy a croissant and a donut $\Diamond(\alpha \vee \beta) \equiv \Diamond(\alpha \wedge \beta)$

Cognitive bias approach

Common assumption: Reasoning and understanding of natural language involve the creation of mental models

- **Understanding** a sentence means being able to mentally construct a model picturing the world which verifies the sentence, and possibly also a model which falsifies it
- **Reasoning** depends on two main processes: first we construct verifying models for the premises and then the validity of the conclusion is checked on these models

Novel hypothesis: biases can constrain the construction of these models and therefore impact both reasoning and interpretations:

- **Neglect-zero** prevents the constructions of zero-models;
- **No-split** expresses a dispreference for split-states.

Comparison with competing accounts[3]

	Ignorance	FC **inference**	**Scalar implicature**	**Conjunctive** *or*
Neo-Gricean	reasoning	reasoning	reasoning	—
Grammatical view	debated	grammatical	grammatical	grammatical
Cognitive bias	neglect-zero	neglect-zero	—	negl-z + no-split

Recent experiments

- Degano, Romoli *et al*, NLS, 2025: Neo-Gricean on **ignorance inference**
- Klochowicz, Schlotterbeck *et al* (SuB 24, CogSci 2025, SuB 2025, XPrag 2025): Nihil vs competitors on disjunction & quantifiers
- Bleotu *et al*: on conjunctive *or*

[3] Neo-Gricean: Horn, Soames, Sauerland, . . . Grammatical view: Chierchia, Fox, Singh *et al*, . . .

Modelling cognitive biases in a team semantics

- Natural language sentences translated into formulas of a classical logic interpreted in a team semantics where also biases can be modeled

Team semantics

- Formulas interpreted wrt a set of points of evaluation (a team) rather than single ones [Hodges 1997; Väänänen 2007]
  - Classical modal logic: $[M = \langle W, R, V \rangle]$

$$M, w \models \phi, \text{ where } w \in W$$

  - Team-based modal logic:

$$M, t \models \phi, \text{ where } t \subseteq W$$

- Two crucial features
  - The empty set is among the possible teams: $\emptyset \subseteq W$
  - Multi-membered teams can model split states

Neglect-zero & no-split tendencies

- Neglect-zero modelled via non-emptiness atom NE which disallows empty teams as possible verifiers

$$M, t \models \mathrm{NE} \text{ iff } t \neq \emptyset$$

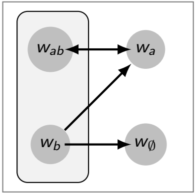
- No-split modelled via flattening operator F which induces pointwise evaluations and therefore avoids simultaneous processing of alternatives

$$M, t \models \mathrm{F}\phi \text{ iff for all } w \in t : M, \{w\} \models \phi$$

BSML: Classical Modal Logic + NE

Language

$$\phi := p \mid \neg\phi \mid \phi \vee \phi \mid \phi \wedge \phi \mid \Diamond\phi \mid \text{NE}$$



Bilateral team semantics

Given a Kripke model $M = \langle W, R, V\rangle$ & states $s, t, t' \subseteq W$

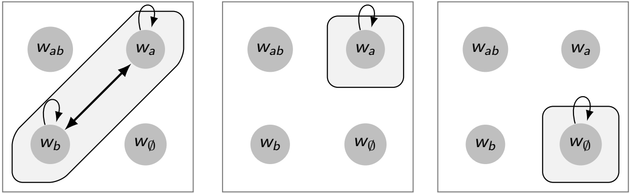
$M, s \models p$	iff	for all $w \in s : V(w, p) = 1$
$M, s \mathrel{=\!\!\mid} p$	iff	for all $w \in s : V(w, p) = 0$
$M, s \models \neg\phi$	iff	$M, s \mathrel{=\!\!\mid} \phi$
$M, s \mathrel{=\!\!\mid} \neg\phi$	iff	$M, s \models \phi$
$M, s \models \phi \vee \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \models \phi$ & $M, t' \models \psi$
$M, s \mathrel{=\!\!\mid} \phi \vee \psi$	iff	$M, s \mathrel{=\!\!\mid} \phi$ & $M, s \mathrel{=\!\!\mid} \psi$
$M, s \models \phi \wedge \psi$	iff	$M, s \models \phi$ & $M, s \models \psi$
$M, s \mathrel{=\!\!\mid} \phi \wedge \psi$	iff	there are $t, t' : t \cup t' = s$ & $M, t \mathrel{=\!\!\mid} \phi$ & $M, t' \mathrel{=\!\!\mid} \psi$
$M, s \models \Diamond\phi$	iff	for all $w \in s : \exists t \subseteq R[w] : t \neq \emptyset$ & $M, t \models \phi$
$M, s \mathrel{=\!\!\mid} \Diamond\phi$	iff	for all $w \in s : M, R[w] \mathrel{=\!\!\mid} \phi$ [where $R[w] = \{v \in W \mid wRv\}$]
$M, s \models \text{NE}$	iff	$s \neq \emptyset$
$M, s \mathrel{=\!\!\mid} \text{NE}$	iff	$s = \emptyset$

Entailment: $\phi_1, \ldots, \phi_n \models \psi$ iff for all M, s: $M, s \models \phi_1, \ldots, M, s \models \phi_n \Rightarrow M, s \models \psi$

Proof Theory: Anttila et al (2024); Expressive completeness: Anttila & Knudstorp (2025);
Comparisons via translation into Modal Information Logic: Knudstorp, Bezhanishvili et al

Neglect-zero effects in BSML: split disjunction

- A state s supports a **disjunction** $(\alpha \vee \beta)$ iff s is the union of two substates, each supporting one of the disjuncts



(a) Verifier (b) **Zero-model** (c) Falsifier

Figure: Models for $(a \vee b)$.

- $\{w_a\}$ verifies $(a \vee b)$ by virtue of an empty witness for the second disjunct, $\{w_a\} = \{w_a\} \cup \emptyset$ & $M, \emptyset \models b$ [$\mapsto$ **zero-model**]
- Main idea: define neglect-zero enrichments, $[\,]^+$, whose core effect is to rule out such zero-models
- Implementation: $[\,]^+$ defined using NE ($s \models \mathrm{NE}$ iff $s \neq \emptyset$), which models neglect-zero in the logic

BSML: neglect-zero enrichment

Non-emptiness

$\textsc{ne}$ is supported in a state if and only if the state is not empty

$$\begin{array}{lll} M, s \models \textsc{ne} & \text{iff} & s \neq \emptyset \\ M, s =\!\mid \textsc{ne} & \text{iff} & s = \emptyset \end{array}$$

Neglect-zero enrichment

For $\textsc{ne}$-free α, $[\alpha]^+$ defined as follows:

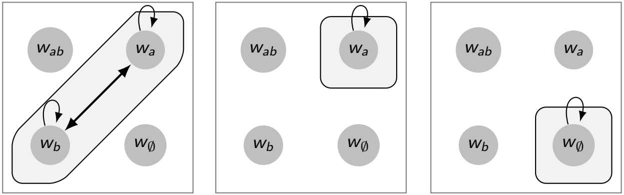
$$\begin{array}{rcl} [p]^+ & = & p \wedge \textsc{ne} \\ [\neg\alpha]^+ & = & \neg[\alpha]^+ \wedge \textsc{ne} \\ [\alpha \vee \beta]^+ & = & ([\alpha]^+ \vee [\beta]^+) \wedge \textsc{ne} \\ [\alpha \wedge \beta]^+ & = & ([\alpha]^+ \wedge [\beta]^+) \wedge \textsc{ne} \\ [\Diamond\alpha]^+ & = & \Diamond[\alpha]^+ \wedge \textsc{ne} \end{array}$$

$[\ ]^+$ enriches formulas with the requirement to satisfy $\textsc{ne}$ distributed along each of their subformulas

Neglect-zero effects in BSML: enriched disjunction

- *s* supports an **enriched disjunction** $[\alpha \vee \beta]^+$ iff *s* is the union of two non-empty substates, each supporting one of the disjuncts

$$[\alpha \vee \beta]^+ = (\alpha \wedge \text{NE}) \vee (\beta \wedge \text{NE}) \wedge \text{NE}$$



(a) $\models [a \vee b]^+$ (b) $\not\models [a \vee b]^+$ (c) $=\!\mid [a \vee b]^+$

- An enriched disjunction requires both disjuncts to be live possibilities

(15) It is raining or snowing $\rightsquigarrow_{nz}$ It might be raining and it might be snowing
$[\alpha \vee \beta]^+ \models \Diamond_e \alpha \wedge \Diamond_e \beta$ (where *R* is state-based)

Formal characterization of neglect-zero effects

$\alpha \rightsquigarrow_{nz} \beta$ (β is a neglect-zero effect of α) iff $\alpha \not\models \beta$ but $[\alpha]^+ \models \beta$

Neglect-zero effects in BSML: main results

- In BSML $[\,]^+$-enrichment has non-trivial effect only when applied to *positive* disjunctions[4]
  - $\mapsto$ we derive FC and related effects (for enriched formulas);
  - $\mapsto$ $[\,]^+$-enrichment vacuous under single negation.

After enrichment

- We derive both wide and narrow scope FC inferences:
  - Narrow scope FC: $[\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Double negation FC: $[\neg\neg\Diamond(\alpha \vee \beta)]^+ \models \Diamond\alpha \wedge \Diamond\beta$
  - Wide scope FC: $[\Diamond\alpha \vee \Diamond\beta]^+ \models \Diamond\alpha \wedge \Diamond\beta$ (if R is indisputable)
  - Modal disjunction: $[\alpha \vee \beta]^+ \models \Diamond_e\alpha \wedge \Diamond_e\beta$ (if R is state-based)
- while no undesirable side effects obtain with other configurations:
  - Dual prohibition: $[\neg\Diamond(\alpha \vee \beta)]^+ \models \neg\Diamond\alpha \wedge \neg\Diamond\beta$

Before enrichment

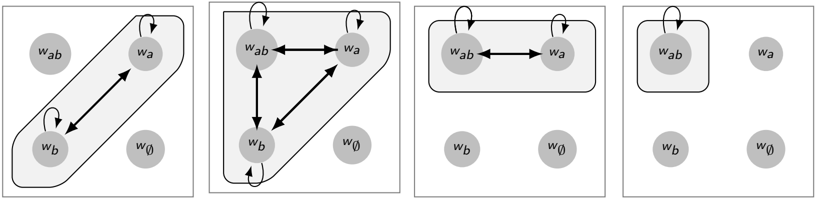
- The NE-free fragment of BSML is equivalent to classical modal logic (ML):
  $$\alpha \models_{BSML} \beta \text{ iff } \alpha \models_{ML} \beta \qquad [\text{if } \alpha, \beta \text{ are NE-free}]$$
  [if α is NE-free: $M, s \models \alpha$ iff for all $w \in s : M, \{w\} \models \alpha$]
- But we can capture the infelicity of epistemic contradictions [Yalcin, 2007] by putting team-based constraints on the accessibility relation:
  1. Epistemic contradiction: $\Diamond_e\alpha \wedge \neg\alpha \models \bot$ (if R is state-based)
  2. Non-factivity: $\Diamond_e\alpha \not\models \alpha$

[4] MA (2022) Logic and Conversation: the case of free choice. *Semantics and Pragmatics* 15(5).

Formal characterization zero and no-zero models

(M, s) is a zero-model for α iff $M, s \models \alpha$, but $M, s \not\models [\alpha]^+$
(M, s) is a no-zero verifier for α iff $M, s \models [\alpha]^+$

Many no-zero verifiers for enriched disjunction



(d) no-zero & scalar $\models \neg(a \wedge b)$

(e) no-zero, non-scalar $\not\models \neg(a \wedge b)$

(f) no-zero, non-scalar & no-uncertain $\not\models \neg\Box_e a$

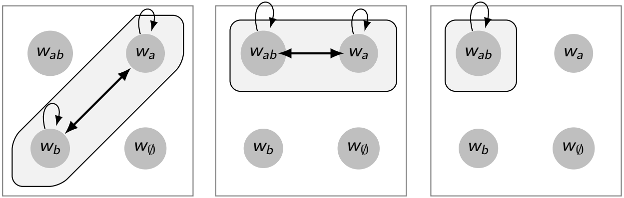
(g) no-zero, non-scalar, no-uncertain & no-split $\models (a \wedge b)$

Figure: Models for enriched $[a \vee b]^+$.

1. Neglect-zero enrichment does not derive scalar implicatures;
2. Neglect-zero enrichment does not derives uncertain inferences $\mapsto$ in contrast to standard neo-Gricean approach to ignorance $\Leftarrow$
3. No-split verifiers compatible with neglect-zero enrichments
   - **No-split** conjecture: only no-split verifiers accessible to ‘conjunctive’ pre-school children. [Klochowicz, Sbardolini, MA. SuB, 2025]

Neglect-zero effects in BSML: possibility vs uncertainty

- More no-zero verifiers for $a \vee b$:



(a) scalar (b) no-uncertain (c) no-split

- Two components of full ignorance ('speaker doesn't know which'):[5]

(16) It is raining or it is snowing $(\alpha \vee \beta) \rightsquigarrow$

a. Uncertainty: $\neg\Box_e\alpha \wedge \neg\Box_e\beta$

b. Possibility: $\Diamond_e\alpha \wedge \Diamond_e\beta$ (equiv $\neg\Box_e\neg\alpha \wedge \neg\Box_e\neg\beta$)

- **Fact:** Only possibility derived as neglect-zero effect:
  - $[a \vee b]^+ \models \Diamond_e a \wedge \Diamond_e b$, but $[a \vee b]^+ \not\models \neg\Box_e a \wedge \neg\Box_e b$ (R is state-based)
  - $\{w_{ab}, w_a\} \models [a \vee b]^+$, but $\not\models \neg\Box_e a$
  - $\{w_{ab}\} \models [a \vee b]^+$, but $\not\models \neg\Box_e a$; $\not\models \neg\Box_e b$

[5] Degano, Marty, Ramotowska, MA, Breheny, Romoli, Sudo. *Nat Lang Sem*, 2025.

Two derivations of full ignorance

1. Standard neo-Gricean derivation [Sauerland 2004]

   (i) Uncertainty derived through quantity reasoning

   (17) $\alpha \vee \beta$ ASSERTION

   (18) $\neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY (from QUANTITY)

   (ii) Possibility derived from uncertainty and quality about assertion

   (19) $\Box_e(\alpha \vee \beta)$ QUALITY ABOUT ASSERTION

   (20) $\Rightarrow \Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY

2. Neglect-zero derivation

   (i) Possibility derived as neglect-zero effect

   (21) $\alpha \vee \beta$ ASSERTION

   (22) $\Diamond_e\alpha \wedge \Diamond_e\beta$ POSSIBILITY (from NEGLECT-ZERO)

   (ii) Uncertainty derived from possibility and scalar reasoning

   (23) $\neg(\alpha \wedge \beta)$ SCALAR IMPLICATURE

   (24) $\Rightarrow \neg\Box_e\alpha \wedge \neg\Box_e\beta$ UNCERTAINTY

Neo-Gricean vs neglect-zero explanation

Contrasting predictions of competing accounts of ignorance

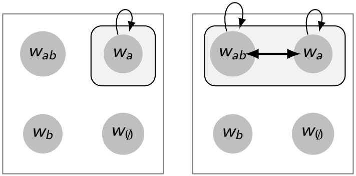
- Neo-Gricean: No possibility without uncertainty
- Neglect-zero: Possibility derived independently from uncertainty

Experimental findings [Degano, Romoli *et al* 2025]

- Using adapted mystery box paradigm, compared conditions in which
  - both uncertainty and possibility are false [**zero-model**]
  - uncertainty false but possibility true [no-zero, no-uncertain model]
- Less acceptance when possibility is false (95% vs **44%**)

⇒ Evidence that possibility can arise without uncertainty

- A challenge for the traditional neo-Gricean approach



(d) **zero-model** (e) no-uncertain

Figure: Models for $(a \vee b)$

Neglect-zero effects on quantifiers

- So far focus on disjunction (propositional BSML)
- NEXT: neglect-zero effects on quantifiers (first order qBSML$^{\rightarrow}$)[6]
- Same methodology (summarized below) but now we work with a first order language and teams are defined as sets of world-assignment pairs
  - $M, s \models \varphi \rightarrow \psi$ iff there is $t \subseteq s : M, t \models \varphi$ & for all $t \subseteq s : M, t \models \phi \Rightarrow M, t \models \psi$ [Priest 1999]
  - $M, s \dashv \phi \rightarrow \psi$ iff for all $w \in s$: $M, \{w\} \models \phi$ & $M, \{w\} \not\models \psi$

Summary neglect-zero effects in team semantics

- Natural language sentences translated into classical logic formulas α
- Logical language interpreted in a team semantics where we can model neglect-zero (via NE)

α : literal meaning $[\alpha]^+$: neglect-zero enriched meaning

- Formal characterisation of zero-models and neglect-zero effects:
  - A zero-model for α is one which verifies α but does not verify $[\alpha]^+$

$$(M, t) \text{ zero-model for } \alpha \text{ iff } M, t \models \alpha \text{ but } M, t \not\models [\alpha]^+$$

  - β is a neglect-zero effect of α iff β follows only if we rule out possible zero-models of α:

$$\alpha \rightsquigarrow_{nz} \beta \text{ iff } \alpha \not\models \beta \text{ but } [\alpha]^+ \models \beta$$

[6] MA & vOrmondt, Modified numerals and split disjunction. *J of Log Lang and Inf* (2023)

Neglect-zero effects on quantifiers: Empty Set (ES) inferences

Predictions of qBSML$^{\rightarrow}$

(25) Less than three squares are black $\mapsto \forall xyz((Sx \wedge Bx \wedge \dots) \rightarrow (x = y \vee \dots))$

a. Verifier: [■, □, ■]
b. Falsifier: [■, ■, ■]
c. Zero-models: [□, □, □]; [▲, ▲, ▲]; . . . $\rightsquigarrow_{nz}$ there are black squares

(26) Every square is black. $\mapsto \forall x(Sx \rightarrow Bx)$

a. Verifier: [■, ■, ■]
b. Falsifier: [■, □, ■]
c. Zero-models: [△, △, △]; [▲, ▲, ▲]; . . . $\rightsquigarrow_{nz}$ there are squares

(27) No squares are black. $\mapsto$ (i) $\forall x(Sx \rightarrow \neg Bx)$; (ii) $\neg\exists x(Sx \wedge Bx)$

a. Verifier: [□, □, □]
b. Falsifier: [■, □, □]
c. Zero-models for (i): [△, △, △]; [▲, ▲, ▲]; . . . $\rightsquigarrow_{nz}$ there are squares
d. Zero-models for (ii): none — no neglect-zero effect

(28) Every square is red or white. $\mapsto \forall x(Sx \rightarrow (Rx \vee Wx))$

a. Verifier: [■ (red), □, ■ (red)]
b. Falsifier: [■ (red), □, ■]
c. Zero-models: [■ (red), ■ (red), ■ (red)]; [□, □, □]; . . . $\rightsquigarrow_{nz}$ there are white & red squares

These predictions tested in Bott, Klochowicz, Schlotterbeck *et al* (2024, 2025)

Experimenting with quantifiers and disjunction

Four non-classical interpretations

(29) a. Some of the squares are black ⇒ not all of the squares are black [UB]
b. Each square is red or white ⇒ there are white squares and red squares [DIST]
c. Less than 3 squares are black ⇒ there are some black squares [ES-scope]
d. Less than 3/every/no squares are black ⇒ there are some squares [ES-restrictor]

Three competing accounts

	UB	DIST	ES-scope	ES-restrictor
Alternative-based	implicature	implicature	implicature	implicature
Bott *et al*, 2019	—	—	neglect-zero	presupposition
Nihil	—	neglect-zero	neglect-zero	neglect-zero

Two experiments

- Exp 1: Answering questions about the emptyset (O. Bott *et al*, SuB 2024)
- Exp 2: Priming with zero-models (Klochowicz, Schlotterbeck *et al*, CogSci 2025, SuB 2025)

Three main conclusions

1. Evidence that ES-restrictor is a presupposition (Exp 1)
2. Evidence that UB differs from both ES-scope and DIST (Exp1 and Exp2)
3. Some evidence that ES-scope and DIST involve the same cognitive process (Exp 2)

Conclusions

- FC, possibility, ES, DIST: a mismatch between logic and language
- Grice's insight:
  - stronger meanings can be derived paying more "attention to the nature and importance to the conditions governing conversation"
- Nihil proposal: some non-classical inferences due to cognitive bias rather than Gricean reasonings
  - FC, possibility, ES, DIST and related inferences as neglect-zero effects

    Literal meanings (classical fragment) + cognitive factor (NE) $\Rightarrow$ FC, possibility, ES-scope, DIST, etc

  - Conjunctive *or* as no-zero + no-split effect

    Literal meanings (classical fragment) + cognitive factors (NE, F) $\Rightarrow$ conjunctive *or*

- Implementation in (extensions of) BSML, a team-based modal logic
- Recent experiments provide some evidence in agreement with the neglect-zero hypothesis, but much more needed

Collaborators & related (future) research

Logic

Proof theory (Anttila, Yang); expressive completeness (Anttila, Knudstorp); bimodal perspective (Knudstorp, Baltag, van Benthem, Bezhanishvili); qBSML (van Ormondt); BiUS & qBiUS (MA); typed BSML (Muskens); connexive logic (Knudstorp, Ziegler & MA); belief revision (Klochowicz) ...

Language

FC cancellations (Pinton, Hui); modified numerals (vOrmondt); attitude verbs (Yan); conditionals (Flachs, Ziegler); questions (Klochowicz); quantifiers (Klochowicz, Bott, Schlotterbeck); indefinites (Degano); homogeneity (Sbardolini); acquisition (Klochowicz, Sbardolini); experiments (Degano, Klochowicz, Ramotowska, Bott, Schlotterbeck, Marty, Breheny, Romoli, Sudo, Szymanik, Visser); ...

THANK YOU![7]

[7] This work was supported by NWO OC project *Nothing is Logical* (grant no 406.21.CTW.023).

BSML & related systems: information states vs possible worlds

- Failure of bivalence in BSML

$$M, s \not\models p \ \& \ M, s \not\models \neg p, \text{ for some info state } s$$

- Info states: less determinate than possible worlds
  - just like truthmakers, situations, possibilities, . . .
- Technically:
  - Truthmakers/possibilities: points in a partially ordered set
  - Info states: sets of possible worlds, also elements of a partially ordered set, the Boolean lattice $Pow(W)$
- Thus systems using these structures are closely connected, although might diverge in motivation:
  - Truthmaker & possibility semantics: description of ontological structures in the world
  - BSML & inquisitive semantics: explaining patterns in inferential & communicative human activities
- NEXT:
  - Comparison via translations in Modal Information Logic [vBenthem19]

BSML & related systems: comparisons via translation

- Modal Information Logic (MIL) (van Benthem, 1989, 2019):[8] common ground where related systems can be interpreted and their connections and differences can be explored
- Goal: translations into MIL of the following systems:
  - BSML
  - Truthmaker semantics (Fine)
  - Possibility semantics (Humberstone, Holliday)
  - Inquisitive semantics (Ciardelli, Groenendijk & Roelofsen)

  (cf. Gödel's (1933) translation of intuitionistic logic into modal logic)
- Here focus on propositional fragments
  - disjunction
  - negation
- (Based on work in progress with Søren B. Knudstorp, Nick Bezhanishvili, Johan van Benthem and Alexandru Baltag)

[8] Johan van Benthem (2019) Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic*.

Modal Information Logic (MIL)

Language

$$\phi \quad ::= \quad p \mid \neg\phi \mid \phi \wedge \phi \mid \phi \vee \phi \mid \langle sup \rangle \phi\psi$$

where $p \in A$.

Models and interpretation

Formulas are interpreted on triples $M = (X, \leq, V)$ where $\leq$ is a partial order

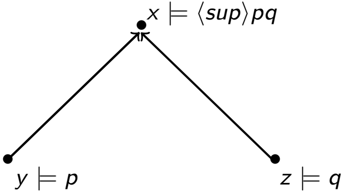
$$\begin{array}{rcl} \mathcal{M}, x \models p & \text{iff} & x \in V(p) \\ \mathcal{M}, x \models \neg\phi & \text{iff} & \mathcal{M}, x \not\models \phi \\ \mathcal{M}, x \models \phi \wedge \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ and } \mathcal{M}, x \models \psi \\ \mathcal{M}, x \models \phi \vee \psi & \text{iff} & \mathcal{M}, x \models \phi \text{ or } \mathcal{M}, x \models \psi \\ \mathcal{M}, x \models \langle sup \rangle \phi\psi & \text{iff} & \text{there are } y, z : x = sup_{\leq}(y, z) \;\&\; \mathcal{M}, y \models \phi \;\&\; \mathcal{M}, z \models \psi \end{array}$$

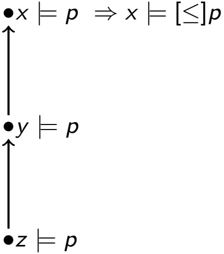
$[\leq]\phi = \neg\langle sup \rangle(\neg\varphi)\top$

$$\mathcal{M}, x \models [\leq]\phi \quad \text{iff} \quad \text{for all } y : y \leq x \;\Rightarrow\; \mathcal{M}, y \models \phi$$

Modal Information Logic (MIL)

Examples





Translations into Modal Information Logic

- Possibility semantics (Humberstone, Holliday)[9]

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & [\leq]\langle\leq\rangle(tr(\phi) \vee tr(\psi)) \\ & \vdots & \end{array}$$

- Inquisitive semantics (Groenendijk, Roelofsen and Ciardelli)

$$\begin{array}{rcl} & \vdots & \\ tr(\neg\phi) & = & [\leq]\neg tr(\phi) \\ tr(\phi \wedge \psi) & = & tr(\phi) \wedge tr(\psi) \\ tr(\phi \vee \psi) & = & tr(\phi) \vee tr(\psi) \\ & \vdots & \end{array}$$

[9] Johan van Benthem, Nick Bezhanishvili, Wesley H. Holliday, A bimodal perspective on possibility semantics, *Journal of Logic and Computation*, Volume 27, Issue 5, July 2017, Pages 1353–1389.

Translations into Modal Information Logic

- Truthmaker semantics (Fine): $\leq$ is "part of" relation[10]

$$\begin{aligned}
&\dots \\
(\neg\phi)^+ &= (\phi)^- \\
(\neg\phi)^- &= (\phi)^+ \\
(\phi \vee \psi)^+ &= (\phi)^+ \vee (\psi)^+ \\
(\phi \vee \psi)^- &= \langle sup \rangle (\phi)^- (\psi)^- \\
(\phi \wedge \psi)^+ &= \langle sup \rangle (\phi)^+ (\psi)^+ \\
(\phi \wedge \psi)^- &= (\phi)^- \vee (\psi)^-
\end{aligned}$$

- BSML: $\leq$ is subset relation $\subseteq$, . . .

$$\begin{aligned}
&\dots \\
(\neg\phi)^+ &= (\phi)^- \\
(\neg\phi)^- &= (\phi)^+ \\
(\phi \vee \psi)^+ &= \langle sup \rangle (\phi)^+ (\psi)^+ \\
(\phi \vee \psi)^- &= (\phi)^- \wedge (\psi)^- \\
(\phi \wedge \psi)^+ &= (\phi)^+ \wedge (\psi)^+ \\
(\phi \wedge \psi)^- &= \langle sup \rangle (\phi)^- (\psi)^- \\
&\dots
\end{aligned}$$

Goal: with **0** (classical modal logic);[11] without **0** (BSML*).

[10] van Benthem, Implicit and Explicit Stances in Logic, *Journal of Philosophical Logic* (2019).

[11] Humberstone, Operational Semantics for Positive R. *Notre Dame J of Form Log* (1988).

Disjunction and Negation

- Three notions of disjunction expressible in MIL:
  - Boolean disjunction: $\phi \vee \psi$
    [classical logic, intuitionistic logic, inquisitive logic]
  - Lifted/tensor/split disjunction: $\langle sup \rangle \phi\psi$
    [BSML, dependence logic, team semantics, operational semantics for Positive R]
  - Cofinal disjunction: $[co](\phi \vee \psi)$ (where $[co]\phi =: [\leq]\langle\leq\rangle\phi$)
    [possibility semantics, dynamic semantics]
- Three notions of negation:
  - Boolean negation: $\neg\phi$
    [classical logic, ...]
  - Bilateral negation: $(\neg\phi)^+ = (\phi)^-$ & $(\neg\phi)^- = (\phi)^+$
    [truthmaker semantics, BSML, ...]
  - Intuitionistic-like negation: $[\leq]\neg\phi$
    [possibility semantics, inquisitive semantics, intuitionistic logic]
- Some combinations:
  - Boolean disjunction + boolean negation $\mapsto$ classical logic
  - Boolean notions in other combinations can generate non-classicality:
    - Boolean disjunction + intuitionistic negation $\mapsto$ intuitionistic logic
  - Classicality also generated by non-boolean combinations:
    - Split disjunction + bilateral negation (classical fragm. BSML)

Experimenting with quantifiers and disjunction

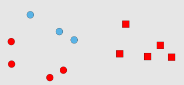
Non-classical interpretations

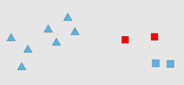
(30)
a. Some of the squares are black ⇒ not all of the squares are black [UB]
b. Each square is red or white ⇒ there are white squares and red squares [DIST]
c. Less than 3 squares are black ⇒ there are some black squares [ES-scope]
d. Less than 3/every/no squares are black ⇒ there are some squares [ES-restrictor]

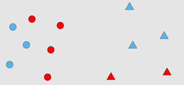
Exp1: Bott et al, SuB 2024

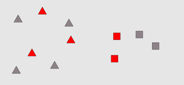
- Question-answer task:

(31) Ist jedes Dreieck entweder rot oder blau? Ja/Nein/Komische Frage
(Is every triangle either red or blue?) Yes/No/Odd question


empty restr


DIST target (zero-model)


control '*yes*'


control '*no*'

- Main results:
  1. Evidence that ES-restrictor is a presupposition: questions in empty restrictor models uniformly perceived as odd
  2. ES-scope (37%) and DIST (23%) unaffected by question environment; UB much less available (10%, while 40% when unembedded)
  3. Inconclusive evidence on whether ES-scope and DIST had the same source

Experimenting with quantifiers and disjunction

Non-classical interpretations

(32) a. Some of the squares are black ⇒ not all of the squares are black [UB]
b. Each square is red or white ⇒ there are white and red squares [DIST]
c. At most 2 squares are black ⇒ there are some black squares [ES-scope, sup]
d. Less than 3 squares are black ⇒ there are some black squares [ES-scope, comp]

Three competing accounts

	UB	DIST	ES-scope	ES-restrictor
Alternative-based	implicature	implicature	implicature	implicature
Bott et al 2019	—	—	neglect-zero	presupposition
Nihil	—	neglect-zero	neglect-zero	neglect-zero

Exp2: Klochowicz, Schlotterbeck *et al*, CogSci 2025, SuB 2025

- Tested whether frequency of strengthening in (32-d) changed after participants were primed to suspend other strengthenings in (32-a-c).
- Results:
  1. Semantic priming between DIST and ES-scope
  2. No priming between UB and ES-scope
  3. No trial-to-trial priming from ES-scope (sup) to ES-scope (com) but spill-over and adaptation effects

qBSML: Quantified Modal Logic + NE

Language:

$t ::= c | v$

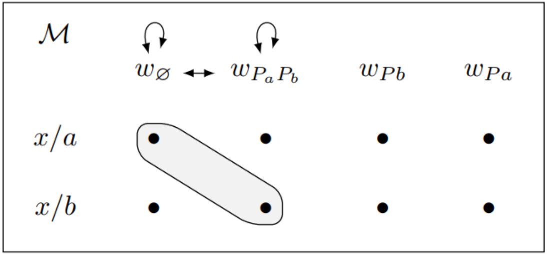
$\phi ::= P^n(\vec{t}) \mid \neg\phi \mid \phi \vee \psi \mid \phi \wedge \psi \mid \exists v\phi \mid \forall v\phi \mid \Box\phi \mid \text{NE}$

Model:

$\mathcal{M} = \langle W, D, R, I \rangle$

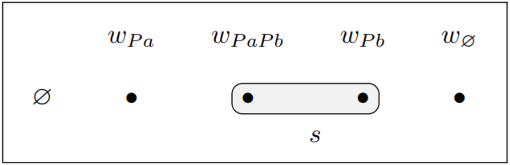
Information State:

A state is set of indices $i = \langle w_i, g_i \rangle$, where $w_i \in W$ and g_i is a variable assignment function



Example of an information state

Empty assignment



A state with an empty assignment

What happens when a variable is added to such information state?

Operations on States

x-extension of an assignment:

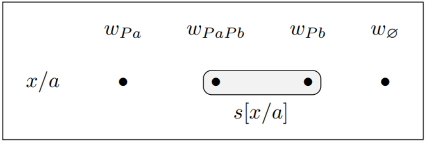
$$g[x/d] := (g \setminus \{\langle x, g(x)\rangle\}) \cup \{\langle x, d\rangle\}$$

Individual x-extension of an index:

$$i[x/d] := \langle w_i, g_i[x/d]\rangle$$

Individual x-extension of a state:

$$s[x/d] := \{i[x/d] | i \in s\}$$

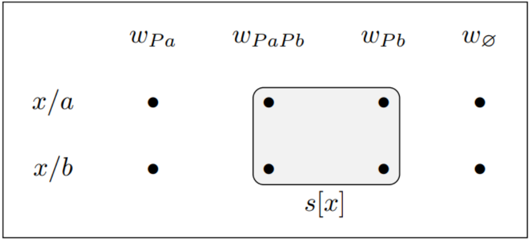


Individual x-extension

Operations on States

Universal x-extension:

$$s[x] := \{i[x/d] | i \in s \; \& \; d \in D\}$$



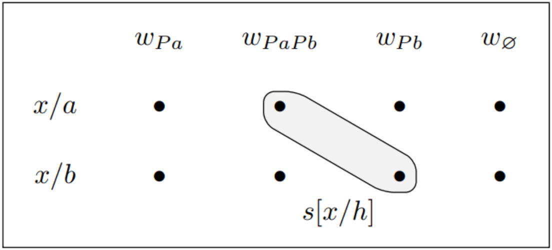
Universal x-extension

Assume $D = \{a, b\}$

Operations on States

Functional *x*-extension:

$$s[x/h] := \{i[x/d] | i \in s \ \& \ d \in h(i)\}$$
$$h : s \mapsto \wp(D) \backslash \varnothing$$



Functional *x*-extension

Semantic Clauses

$$
\begin{array}{lll}
\mathcal{M}, s \models P^n t_1 \ldots t_n & \text{iff} & \forall i \in s : \langle [t_1]_{\mathcal{M},i}, \ldots, [t_n]_{\mathcal{M},i} \rangle \in I(w_i)(P^n) \\
\mathcal{M}, s \dashv P^n t_1 \ldots t_n & \text{iff} & \forall i \in s : \langle [t_1]_{\mathcal{M},i}, \ldots, [t_n]_{\mathcal{M},i} \rangle \notin I(w_i)(P^n) \\
\mathcal{M}, s \models \neg\varphi & \text{iff} & \mathcal{M}, s \dashv \varphi \\
\mathcal{M}, s \dashv \neg\varphi & \text{iff} & \mathcal{M}, s \models \varphi \\
\mathcal{M}, s \models \varphi \vee \psi & \text{iff} & \exists t, t' : t \cup t' = s \text{ and } \mathcal{M}, t \models \varphi \text{ and } \mathcal{M}, t' \models \psi \\
\mathcal{M}, s \dashv \varphi \vee \psi & \text{iff} & \mathcal{M}, s \dashv \varphi \text{ and } \mathcal{M}, s \dashv \psi \\
\mathcal{M}, s \models \varphi \wedge \psi & \text{iff} & \mathcal{M}, s \models \varphi \text{ and } \mathcal{M}, s \models \psi \\
\mathcal{M}, s \dashv \varphi \wedge \psi & \text{iff} & \exists t, t' : t \cup t' = s \text{ and } \mathcal{M}, t \dashv \varphi \text{ and } \mathcal{M}, t' \dashv \psi \\
\mathcal{M}, s \models \Box\varphi & \text{iff} & \forall i \in s : \mathcal{M}, R(w_i)[g_i] \models \varphi \\
\mathcal{M}, s \dashv \Box\varphi & \text{iff} & \forall i \in s : \exists X \subseteq R(w_i) \text{ and } X \neq \varnothing \text{ and } \mathcal{M}, X[g_i] \dashv \varphi \\
\mathcal{M}, s \models \mathtt{NE} & \text{iff} & s \neq \varnothing \qquad [X[g_i] = \{\langle w, g_i \rangle \mid w \in X\}] \\
\mathcal{M}, s \dashv \mathtt{NE} & \text{iff} & s = \varnothing \qquad [R(w_i) = \{v \in W \mid w_i R v\}] \\
\mathcal{M}, s \models \forall x\varphi & \text{iff} & \mathcal{M}, s[x] \models \varphi \\
\mathcal{M}, s \dashv \forall x\varphi & \text{iff} & \mathcal{M}, s[x/h] \dashv \varphi, \text{ for some } h : s \to \wp(D) \setminus \varnothing \\
\mathcal{M}, s \models \exists x\varphi & \text{iff} & \mathcal{M}, s[x/h] \models \varphi, \text{ for some } h : s \to \wp(D) \setminus \varnothing \\
\mathcal{M}, s \dashv \exists x\varphi & \text{iff} & \mathcal{M}, s[x] \dashv \varphi
\end{array}
$$