The Goldblatt Translation between Orthologic and KTB, revisited

Rodrigo N. Almeida – ILLC-UvA

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Summary

1. Translations into Classical logic;
2. Orthologic, Ortholattices, Orthospaces.
3. The Goldblatt Translation.
4. Properties of this translation.
5. Some further thoughts and developments.

Classical Translations

Figure 1: Kurt Gödel (1906-1978); Valery Glivenko (1897-1940)

Definition

Given $\;\; 2 \; L_{G\check{e}G}$ we define the double negation translation into $L_{{}^{2}\check{e}G}$, as follows:

1. $\acute{E}(\text{Ş}) \; = \;\; : \; : \text{Ş}$ and $\acute{E}(\, ? \,) \; = \;\; ?$;
2. $\acute{E}(\quad \wedge \quad) = \acute{E}(\quad) \wedge \acute{E}(\quad)$;
3. $\acute{E}(\, : \quad) \; = \;\; : \acute{E}(\quad)$.

Figure 1: Kurt Gödel (1906-1978); Valery Glivenko (1897-1940)

Definition

Given 2 $L_{Gĕ G}$ we define the double negation translation into $L_{²ĕG}$, as follows:

1. $É$(Ş) = : : Ş and $É$(?) = ?;
2. $É$(∧) = $É$( ) ∧ $É$( );
3. $É$(:) = : $É$( ).

Theorem (Glivenko,1929)

{ω ā˙ā ìω g ·ƴh ¯ 2 CPC ȍì hɧă ωɧy ȍì É( ) 2 IPC¡

Figure 2: Alfred Tarski (1901-1983); J.C.C. McKinsey (1908-1953)

Definition

Given $\varphi \in L_{IPC}$ we define the Godel-McKinsey-Tarski (GMT) translation into $S4$, as follows:

1. $GMT(p) = \Box p$ and $GMT(\bot) = \bot$;
2. $GMT(\varphi \wedge \psi) = GMT(\varphi) \wedge GMT(\psi)$ and $GMT(\varphi \vee \psi) = GMT(\varphi) \vee GMT(\psi)$;
3. $GMT(\varphi \to \psi) = \Box(GMT(\varphi) \to GMT(\psi))$.

Godel-McKinsey-Tarski Translation

Figure 2: Alfred Tarski (1901-1983); J.C.C. McKinsey (1908-1953)

Definition

Given $\varphi \in \mathcal{L}_{IPC}$ we define the Godel-McKinsey-Tarski (GMT) translation into $\mathsf{S4}$, as follows:

1. $Tr(p) = \Box p$ and $Tr(\bot) = \bot$;
2. $Tr(\varphi \wedge \psi) = Tr(\varphi) \wedge Tr(\psi)$ and $Tr(\varphi \vee \psi) = Tr(\varphi) \vee Tr(\psi)$;
3. $Tr(\varphi \to \psi) = \Box(Tr(\varphi) \to Tr(\psi))$.

Theorem (Godel,1933, McKinsey-Tarski, 1948)

For every $\varphi \in \mathcal{L}_{IPC}$: $\varphi \in \mathsf{IPC}$ iff $Tr(\varphi) \in \mathsf{S4}$.

Blok-Esakia and Modalisation

In the case of the GMT translation much more is true:

Definition
Let $L \in$ Ext(IPC) and $M \in$ NExt(S4). We say that M is a modal companion of L if:

$$\varphi \in L \iff T(\varphi) \in M.$$

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Theorem (Blok, 1976, Esakia 1976)
[illegible] Ext([illegible]) [illegible] NExt(S4.Grz) [illegible]

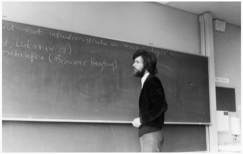

Figure 3: Wim Blok (1947-2003); Leo Esakia (1934-2010)

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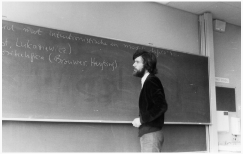

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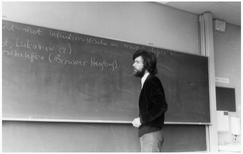
Theorem (Blok, 1976, Esakia 1976)
İȡă̤ă̄ ὂ’ ƕɧ ὂ’ɷ ɡ ɷ̤ʢȡ̊ὂ’ ɡ ƹă̄˝ ă̄ă̄ɧ ¯ȡă ɏƕ¯ὂ!ă̄’ Ext(²ĕ*G*) *ƕɧă* NExt(S4:Grz)˝ *ɡ ƕʢʢὂɧÔ’ ɏɷÔ˥ ὂ!’ ¯ɷ ¯ȡă̄ὂ̤ Ô̤ă̄ƕ¯ă̄’¯ ɡ ɷăƕɏ !ɷ ɡ ʢƕɧὂɷɧ ᶦ*



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This makes the GMT translation very robust, and a very useful tool for the parallel analysis of modal and intuitionistic logic.

More broadly it reflects a vision of non-classical logic as modalised classical logic.

Strong Translations

Slogan: Strong translations preserve and reflect many properties of the systems in question, and can be defined uniformly on extensions.

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Slogan: Strong translations correspond to “smooth transformations” of the classes of models of our logical systems.

Example

In the case of the double negation translation, given a poset $ĕ$ seen as a Kripke frame, the corresponding Boolean model is obtained by looking at Max$(ĕ)$.

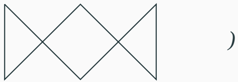
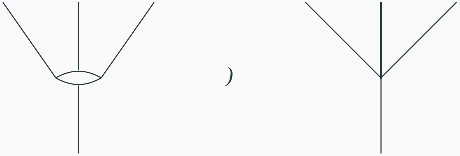


Figure 4: Transformation from Int. Model to Classical Model

Strong Translations (cont.)

Example

In the case of the GMT translation, given a preordered set ě, seen as a transitive and reflexive Kripke frame, the corresponding intuitionistic logic is obtained by taking the ’ɉāɏā¯ωɧ:



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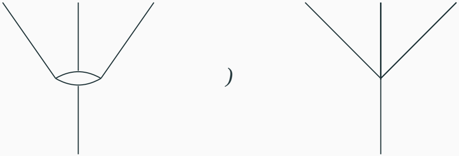

Figure 5: Transformation from S4 model to Int. Model

Intuition: identifying points in a cluster alters only some local properties; erasing worlds destroys global properties.

Orthologic, Ortholattices, Orthospaces

Ortholattices

Definition

An algebra $\mathbf{O} = (A; \wedge; \vee; {}^{\perp}; 0; 1)$ is said to be an ortholattice when $(A; \wedge; \vee; 0; 1)$ is a bounded lattice, and ${}^{\perp}$ satisfies the following properties for every $a; b \in A$:

1. $(a \wedge b)^{\perp} = a^{\perp} \vee b^{\perp}$ and $(a \vee b)^{\perp} = a^{\perp} \wedge b^{\perp}$;
2. $a \wedge a^{\perp} = 0$ and $a \vee a^{\perp} = 1$
3. $(a^{\perp})^{\perp} = a$.

These are the same axioms of Boolean algebras except for distributivity.

Definition

An algebra $\mathbf{O} = (L; \wedge; \vee; ^{\perp}; 0; 1)$ is said to be an ortholattice when $(L; \wedge; \vee; 0; 1)$ is a bounded lattice, and $^{\perp}$ satisfies the following properties for every $x; y \in L$:

1. $(x \wedge y)^{\perp} = x^{\perp} \vee y^{\perp}$ and $(x \vee y)^{\perp} = x^{\perp} \wedge y^{\perp}$;
2. $x \wedge x^{\perp} = 0$ and $x \vee x^{\perp} = 1$
3. $(x^{\perp})^{\perp} = x$.

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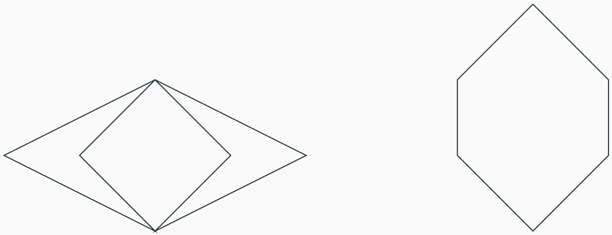

Figure 6: Examples of Ortholattices ($\grave{U}\ddot{e}_3$ and Benzene)

Orthologic

Definition

Let $L_{ë}$ be the language of ortholattices. Let `, be a binary consequence relation in this language. Then we say that ` is an *ω‴¯ɖωŷωÔ0!* if it is closed under uniform substitution, and satisfies the following axioms, for all ; ; 2 $L_{ë}$:

1. For a finite set of formulas , ` if and only if ⋁ `
2. ∧ ` ; ∧ `
3. ` ? ? ; ? ? `
4. ∧ : `
5. If ` and ` , then ` ∧
6. If ` and ` then `
7. If ` then ? ` ?

We denote by *ë* the *g0ŋ0g hŷ ω‴¯ɖωŷωÔ0!*.

Definition

Let $L_{\ddot{e}}$ be the language of ortholattices. Let `, be a binary consequence relation in this language. Then we say that ` is an [illegible] if it is closed under uniform substitution, and satisfies the following axioms, for all ; ; 2 $L_{\ddot{e}}$:

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2. $\wedge$ ` ; $\wedge$ `
3. ` ? ? ; ? ? `
4. $\wedge$: `
5. If ` and ` , then ` $\wedge$
6. If ` and ` then `
7. If ` then ? ` ?

We denote by $\ddot{e}$ the [illegible].

Theorem

[illegible] Ext($\ddot{e}$) [illegible] Var($\ddot{e}$) [illegible]

Definition

Let $(ŧ; \hat{G})$ be a set equipped with a binary reflexive, symmetric relation, such that whenever ʾ $\neq$ ˋ, there is some _ such ʾ $\hat{G}$ _ and $:($ˋ $\hat{G}$ _$)$ or vice-versa. We call this an orthoframe.

Orthoframes and orthomodels

Definition
Let $(X, \perp)$ be a set equipped with a binary reflexive, symmetric relation, such that whenever $x \neq y$, there is some z such that $x \perp z$ and $\neg(y \perp z)$ or vice-versa. We call this an orthoframe.
We write

$$A^{\perp} := \{x : \forall y \in A, \neg(x \perp y)\}$$

for the orthogonal complement of A.
Given an orthoframe $(X, \perp)$, we say that a subset A is [illegible] if $A^{\perp\perp} = A$.

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We write

$$Ŀ^{?} := f’ : 8‘ \; 2 \; Ŀ; : (’ \hat{G} ‘)g$$

for the orthogonal complement of Ŀ.
Given an orthoframe $(ŧ; ?)$, we say that a subset Ŀ is ẅăÔ˷ỵhẅ if $Ŀ^{??} = Ŀ$.

Given an orthoframe $(ŧ; ?)$, a valuation $\hat{S}$: ĕẅϖƫ ! ĜăÔ(ŧ) taking values in the regular subsets of ŧ is called an ϖẅ¯ɖϖ g ϖăặỵ. We write M The Kripke semantics of orthomodels is defined as follows:

1. M ; ’ ƫ iff ’ 2 $\hat{S}$(ƫ);
2. M ; ’ ^ iff M ; ’ and M ; ’ ;
3. M ; ’ ? iff whenever ’ $\hat{G}$‘ then M ; ‘ 1 .

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Theorem (Goldblatt, 1974)
ëẅ¯ȡωɏωÔ0̊! 0̊’ ’ω˵ ɧă ƕɧă !ω g Ꞓɏā¯ā ˝ 0̊¯ȡ ẅā’Ꞓā!¯ ¯ω ωẅ¯ȡωȉẅƕ g ā’ᵢ

Figure 7: Robert Goldblatt (1949-)

KTB := K ϛ ! ϛ ϛ ! ϛ

Definition (Goldblatt Translation)

For 2 $L_{ë}$ we define the Goldblatt translation:

1. ~(ϛ) = ~(ϛ)
2. ~(∧) = ~( ) ∧ ~( )
3. ~($^{?}$) = : ~( )

The Goldblatt Translation

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Theorem

Goldblatt Translation

Theorem

{ϖ ā˙ā ȉϖ g ˯ ɏɐ (;) 2 L ë ˝ ā ɖɐ˙ā ¯ɖɐ¯ 2 O ȉ ɐɧă ϖɧɏ ȉ ~( ) ! ~( ) 2 KTB

Proof.

(Sketch) Given an orthomodel $(\mathfrak{t}; \hat{G}; \hat{S})$, we can see it as a model of KTB, such that $(\mathfrak{t}; ?; \hat{S})$ if and only if $(\mathfrak{t}; \hat{G}; \hat{S})$ ~( ).

Theorem
{ωẅ ā˙ȧẅ ὶωẅ g ˬ ɏƕ (;) 2 L ë ˝ ā ɖƕ˙ā ¯ɖƕ¯ 2 O ȍ̀ì ƕђă ωђɏ˓ ȍ̀ì ~( ) ! ~( ) 2 KTB¡

Proof.
(Sketch) Given an orthomodel $(ɬ; \hat{G}; \hat{S})$, we can see it as a model of KTB, such that $(ɬ; ?; \hat{S})$ if and only if $(ɬ; \hat{G}; \hat{S})$ ~( ).
Conversely, given a model $(ɬ; \hat{G}; \hat{S})$ of KTB we can take a ɏωωƩ˥’ɉ ȧɏā¯ωђ:

$$’ \quad ˓ \quad ( ) \quad 8\_ (’ \hat{G}\_ \quad \$ \quad ˓ \hat{G}\_)$$

Using this we form a quotient ɬ := ɬ= , with a relation [’]$\hat{G}$[˓] if and only if ’ $\hat{G}$˓ , and [’] 2 $\check{S}$(Ʃ) if and only if ’ 2 $\hat{S}$(Ʃ). And we have that $(ɬ ; \hat{G}; \check{S})$ if and only if $(ɬ; \hat{G}; \hat{S})$ ~( ). □

Analysing the Goldblatt Translation

A Disillusionment

Despite early enthusiasm with this logic, this line of work went quiet after a while.

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In the early 2000's, Miyazaki produced a detailed analysis of the translation, hinting at the kind of theory present in the GMT translation.

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Let ë 2 (O) and Í 2 **NExt(**KTB**)**. We say that Í is a KTB-companion of ë if:

(;) 2 ë ( ) ~( ) ! ~( ) 2 Í

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(;) 2 ë ( ) ~( ) ! ~( ) 2 Í

Theorem (Miyazaki, 2004)

İdā ìoyyo˝ ůħÔ doyǎ.

1. *{ow āħ!d Í 2* NExt(KTB) *dāwā ů' ħ yoÔů! ë 2* (O) *'! d dħ Í ů' dā goǎħy !og Şħħůoħ oì ë, dů' ħ''ůÔħ gāħ Şwā'āw·ā' Éwů Şjā !og Şyā āħā' ħΣ yħwů ħħǎ {Ùěⁱ*
2. *{ow āħ!d owdoyoÔů! ë 2* (O) *˝ ů d dā {Ùě dāwā ů' ħ yoÔů! Í 2* NExt(KTB) *'! d dħ Í ů' dā goǎħy !og Şħħůoħ oì ë, dů' ħ''ůÔħ gāħ Şwā'āw·ā' ħΣ yħwů ħħǎ {Ùěⁱ*

In Search of a Blok-Esakia

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The following seems to never have been written down:

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[illegible] Ext(O) [illegible] KTB[illegible]

Proof.
By a classic result in the theory of ortholattices, and a result of Miyazaki, we know that the bottom of the lattices of varieties look as follows:

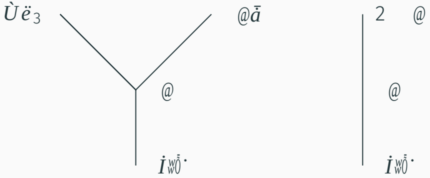

Figure 8: Bottom of the lattice of varieties

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Indeed, unlike in the KTB case, to a single ortholattice there could correspond multiple orthoframes; both of the following are transformed into Benzene frames:

Figure 9: Two Benzene frames

Dashing Hopes (cont'd.)

We were able to show that if this translation was strong – which is measured in categorical terms – then there would need to be a p-morphism between the two previous frames. This does not exist, as can be inspected.

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The formal treatment of this goes through by relating translations with adjunctions. The property which fails above is that the unit of the adjunction is not an isomorphism, i.e., the left adjoint is not fully faithful.

Expanding the Signature of Ortho-lattices

Problems of Duality

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On the other hand, by adding specific implications outside the signature the situation can be made better.

The precise nature of the phenomenon at play is not yet clear to me. But by reverse engineering some well-known situations, I think I can make my point that there is indeed [illegible]

Imagine that we had just discovered pseudocomplemented distributive lattices – bounded distributive lattices of type $\mathtt{D} = (P; \wedge; \vee; \neg; 0; 1)$ where the negation satisfies the following property:

$$\hbar \wedge ! = 0 \iff \hbar \leq \neg !$$

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$$a \wedge b = 0 \iff a \leq \neg b$$

We also found that these could be modelled using posets $(X; \leq)$, in the usual way: taking valuations $V : \mathsf{Prop} \to \mathcal{U}p(X)$, and the semantic clause

$$(X, V), x \Vdash \neg\varphi \iff \forall y \geq x, (X, V), y \nVdash \varphi$$

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We also found that these could be modelled using posets $(W; \leq)$, in the usual way: taking valuations $V : Var \to Up(W)$, and the semantic clause

$$(W; V); w \Vdash \neg\varphi \iff \forall v \geq w; (W; V); v \nVdash \varphi$$

Being experienced modal logicians, we notice that this is very similar to the S4 modal system, and rush to translate this to that system with the following translation:

1. $T(p) = \Box p$ and $T(\bot) = \bot$ and $T(\top) = \top$;
2. $T(\varphi \wedge \psi) = T(\varphi) \wedge T(\psi)$ and $T(\varphi \vee \psi) = T(\varphi) \vee T(\psi)$;
3. $T(\neg\varphi) = \Box\neg T(\varphi)$.

However, we start to notice a few problems:

1. We can define the pre-linearly ordered S4-frames – they are just given by (£ ! t6) _ (t6 ! £) – but these cannot be defined because the natural notion of p-morphism does not preserve linearity!

However, we start to notice a few problems:

1. We can define the pre-linearly ordered S4-frames – they are just given by $(\varphi \to \psi) \lor (\psi \to \varphi)$ – but these cannot be defined because the natural notion of p-morphism does not preserve linearity!
2. You find that the natural semantic transformations – algebraic, duality-theoretic – appear to work in the finite case, but do not extend to the infinite case.

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1. We can define the pre-linearly ordered S4-frames – they are just given by $(\varphi \to \psi) \vee (\psi \to \varphi)$ – but these cannot be defined because the natural notion of p-morphism does not preserve linearity!
2. You find that the natural semantic transformations – algebraic, duality-theoretic – appear to work in the finite case, but do not extend to the infinite case.
3. Eventually you find that there is not isomorphism between the lattices of pseudocomplemented distributive lattices and the extensions of any extension of S4 – the former is countable whilst the latter is of size continuum.

What is wrong with this picture?

The Role of Implication

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Key idea: orthologic as a logic of “mutual consistency”: ’ $\hat{G}$‘ = states at ’ and ‘ are mutually consistent.

This invites a **Kripkean implication** (already alluded to, in some form, by Dalla-Chiara).

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Key idea: orthologic as a logic of “mutual consistency”: ’ Ĝ‘ = states at ’ and ‘ are mutually consistent.

This invites a **Kripkean implication** (already alluded to, in some form, by Dalla-Chiara).

The intended meaning:

ħ , ! ε := \ In all worlds that are consistent with the present, if ħ holds, then ε holds.

This generates a different class of structures, called in my thesis ëẅ¯ɖω0̇ g Ϛŷ0̇!ħ¯0̇˙ā ’‘ ’¯ā g ’.

Adding implications to Goldblatt

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It is left open whether this map is an isomorphism.

Thank you!

Questions?

Some facts obtained about these structures:

1. Their theory is conservative over ortholattices.
2. Every finite ortholattice, and every (infinite-dimensional) Hilbert space, admits the structure of such an implication.
3. The proposed duality allows for great simplification in reasoning (See board).
4. It satisfies a well-defined universal property:

$$! \quad h \; ! \; \varepsilon \quad (\;) \quad h \quad !^{?} \_ \, \varepsilon$$

5. It allows the description of natural objects such as the centre of an orthomodular lattice; this defines a sound translation from classical logic into orthoimplicative logic.
6. Some recent connections: in the case of atomistic ortholattices, the duality we introduced restricts to a known class of graphs called stiff graphs.