Concurrent NetKAT

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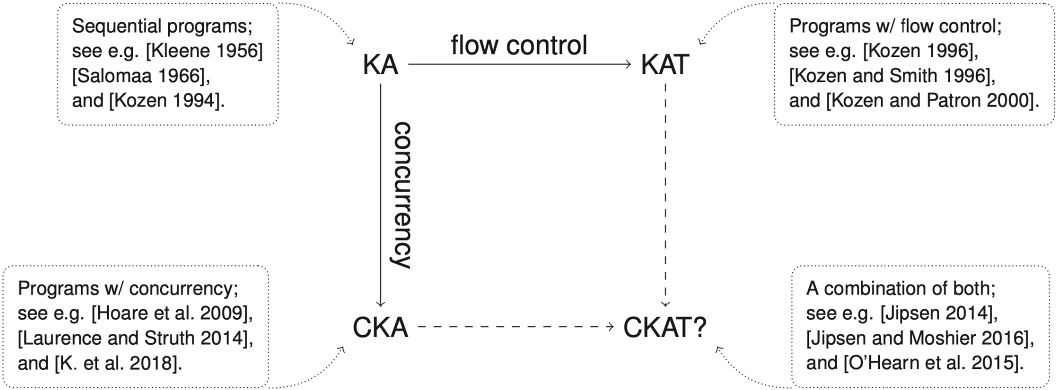
- Key limitation of NetKAT: stateless and sequential


Sequential programs;
see e.g. [Kleene 1956]
[Salomaa 1966],
and [Kozen 1994].
KA
flow control
KAT
Programs w/ flow control;
see e.g. [Kozen 1996],
[Kozen and Smith 1996],
and [Kozen and Patron 2000].
concurrency
Programs w/ concurrency;
see e.g. [Hoare et al. 2009],
[Laurence and Struth 2014],
and [K. et al. 2018].
CKA
CKAT?
A combination of both;
see e.g. [Jipsen 2014],
[Jipsen and Moshier 2016],
and [O'Hearn et al. 2015].

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$$\begin{tikzcd}
\mathrm{KA} \arrow[r, "\text{observations}"] \arrow[d, "\text{concurrency}"] & \mathrm{KAO} \arrow[d, dashed] \\
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$\Rightarrow$ POCKA: fine-grained reasoning about concurrent programs with conditionals that manipulate a shared global memory.

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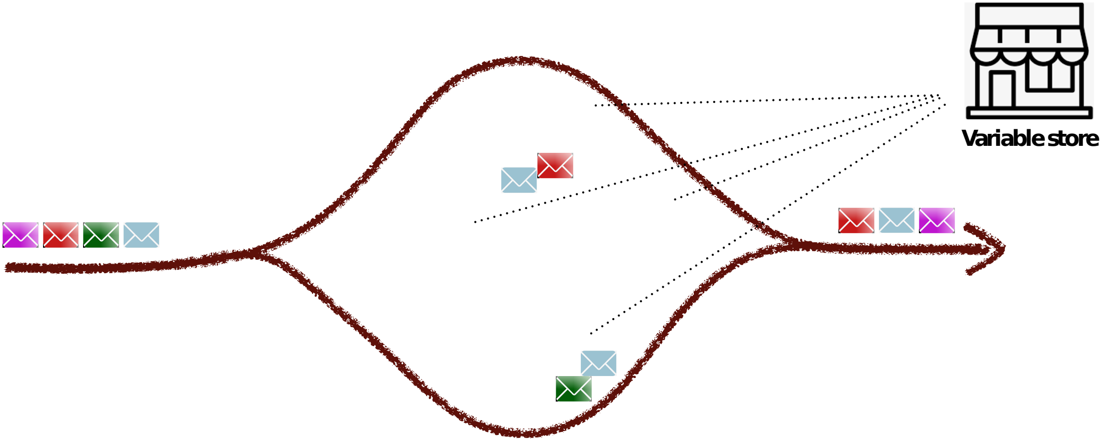
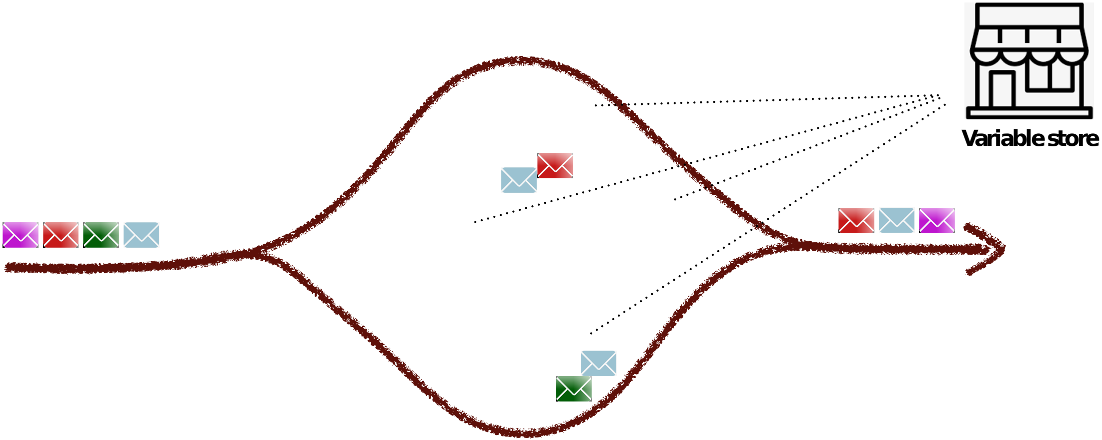
- Key limitation of NetKAT: stateless and sequential
- This paper: Concurrent NetKAT, combination of POCKA and a multi-packet extension of NetKAT
  - Sound and complete axiomatisation of CNetKAT
  - Examples of applicability of CNetKAT for modelling and analysing concurrent network behaviours


Variable store



Do we always see a green packet at switch 3 before we see a red packet at switch 2?

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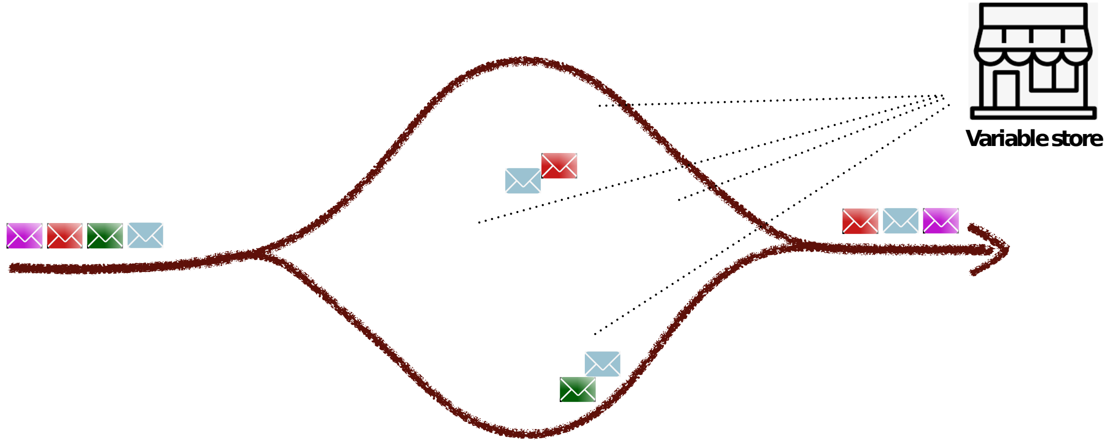
More complicated programs with $\quad + \quad ; \quad * \quad \|$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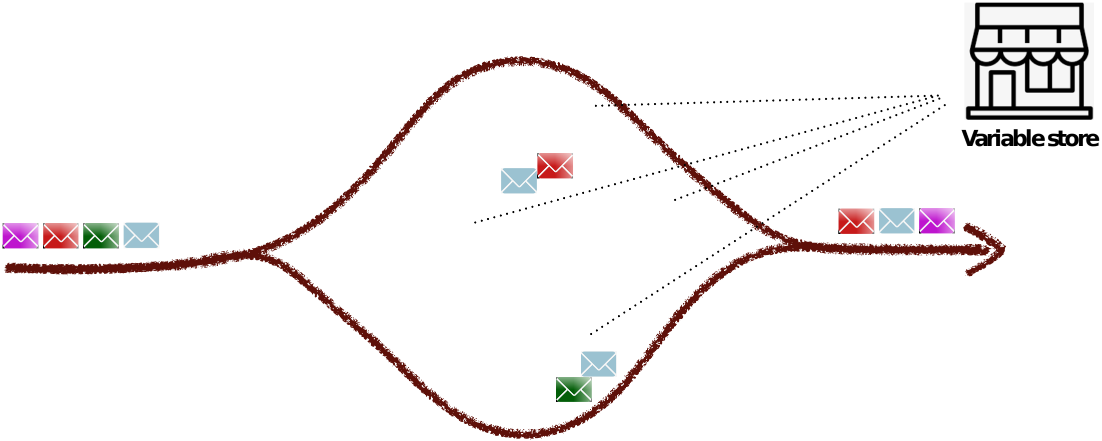
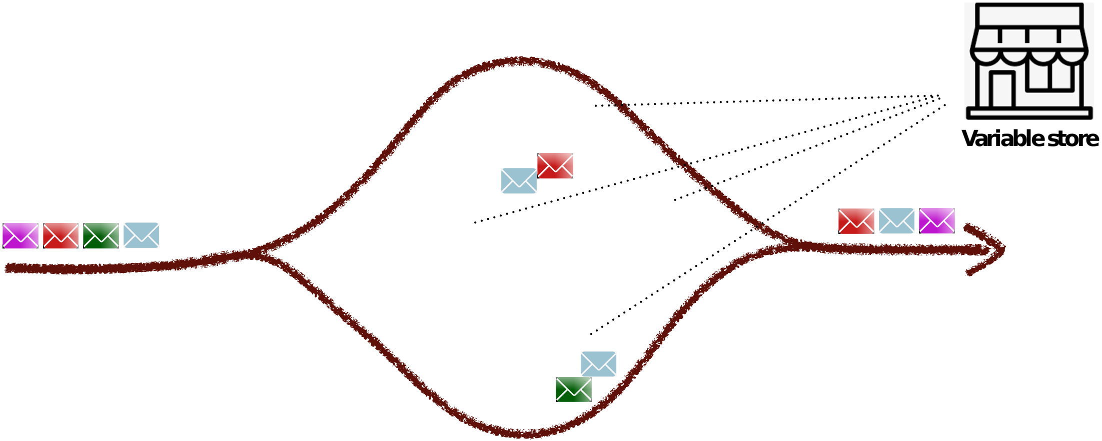
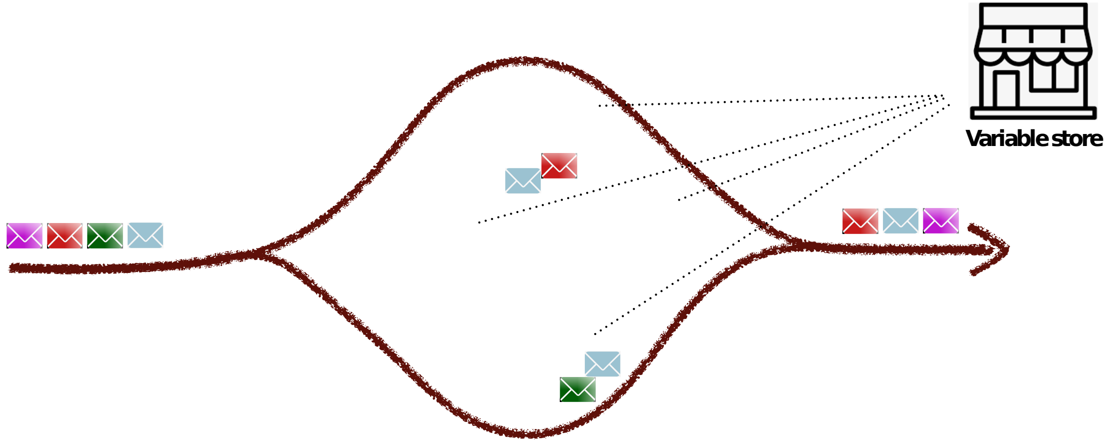


$$p_1 := \mathsf{sw}{=}1 \,;\, ((\mathsf{tag}{=}\textcolor{red}{\mathit{red}} \,;\, \mathsf{sw}{\leftarrow}2) \parallel (\mathsf{tag}{=}\textcolor{green}{\mathit{green}} \,;\, \mathsf{sw}{\leftarrow}3))$$



$$\mathrm{sw}{=}1\,;\,((v{=}1\,;\,\mathrm{tag}{=}\mathit{red}\,;\,\mathrm{sw}{\leftarrow}2)\parallel(\mathrm{tag}{=}\mathit{green}\,;\,\mathrm{sw}{\leftarrow}3\,;\,v{\leftarrow}1))$$



$$p \triangleq v \leftarrow 0 \; ; (p_1 \parallel p_2 \parallel p_3 \parallel p_4)^*$$

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$$[\![p \parallel q]\!](a) \triangleq \{(\mathbf{u} \parallel \mathbf{v}) \cdot (b \cup c) \mid \mathbf{u} \cdot b \in [\![p]\!](a), \mathbf{v} \cdot c \in [\![q]\!](a)\}$$

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Full semantics: $[\![p]\!]{\downarrow}(a)$

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- Axioms connecting the local and the global state

$$\mathsf{drop} \parallel p = p \qquad\qquad \mathsf{abort} \parallel p = \mathsf{abort}$$

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$$t \vee t' = t \parallel t' \qquad o \vee o' = o + o'$$

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$[\![\overline{v = n}]\!]$ should contain all partial functions that assign a value to v different than n.

Theorem

Let $p, q \in \mathrm{Prg}$. We have CNetKAT $\vdash p = q$ if and only if $[\![p]\!]\!\downarrow = [\![q]\!]\!\downarrow$.

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3. Using completeness of POCKA, obtain completeness for programs of the form $s \mathbin{;} \Pi_a$ (and sums thereof), where s is a state program.
4. Lastly, we combine these results to prove that if p and q have the same behaviour on input a, the program $\Pi_a \mathbin{;} p$ is provably equivalent to $\Pi_a \mathbin{;} q$. We can conclude using the extensionality axiom: $\forall a \in 2^{\mathsf{Pk}}.(\Pi_a \mathbin{;} p = \Pi_a \mathbin{;} q) \Rightarrow p = q$.

$$v \leftarrow 1 \,;\, v = 2$$

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$$(v \leftarrow 1\,;\, v = 2) \parallel v \leftarrow 2$$

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How to study isolated behaviour? –> guarded pomsets

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- Future work: lots of more case studies